LivePhoto Physics Homework

Name:______________________

Date:______________________

Using RC Decay to Determine a Capacitance

Figure 1: A competitor's *Mobile Q™ Silver Vehicle Tracking Device* (2005).

A Challenge: You are designing a power supply for a new GPS vehicle-tracking device to improve vehicle safety and theft recovery. You need to complete your design as soon as possible. You only have one suitable capacitor in your lab, and you suddenly realize that you need to have an accurate knowledge of its capacitance. Unfortunately, you have no capacitance or resistance meters.

But, you do have a collection of 5 carbon resistors whose values are marked with color bands and are accurate to within ±5%. In addition you have a battery and two types of voltage measuring devices. One is a conventional analog meter (V_A) and the other is a computer-based voltage sensor (V_C).

You quickly realize that you can use measurements of the change in the potential difference across a charged capacitor (C) placed in a circuit with a carbon resistor (R) to obtain an accurate value of capacitance. You start by reviewing your theoretical knowledge of RC circuits.

Theoretical Review:

[A] A capacitor is *defined* as any two conductors, separated by an insulator where each conductor carries a net excess charge that is equal in magnitude and opposite in sign. Its capacitance, C, is defined as

$$C \equiv \frac{Q}{V} \qquad \text{[Eq. 1]}$$

where Q is the magnitude of the excess charge on each plate and V is the voltage (or potential difference) across the plates.

[B] When a charged capacitor with a voltage V is discharged through a carbon resistor of resistance R, its voltage changes are characterized by an exponential decay given by

$$V = V_o e^{-\left(\frac{1}{RC}\right)t} \qquad \text{[Eq. 2]}$$

where V_0 is the voltage to which it was initially charged.

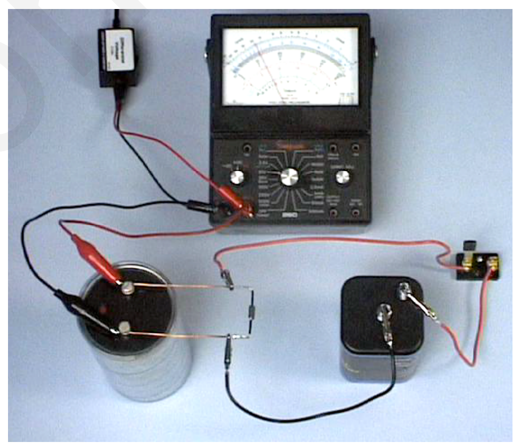


Figure 2: A large capacitor is wired so it is charged by a battery when a switch is closed. Opening the switch allows the capacitor to discharge through a resistor. An analog voltmeter and computer voltage sensor are both wired to monitor the discharge.

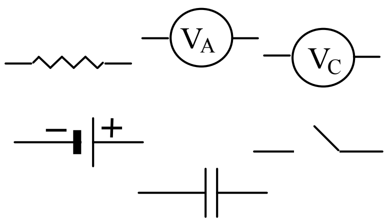
The factor RC in Equation 2 determines the rate of "decay" of the charge and voltage across the capacitor. It is commonly denoted by t_c and called the *time constant* for an RC circuit.

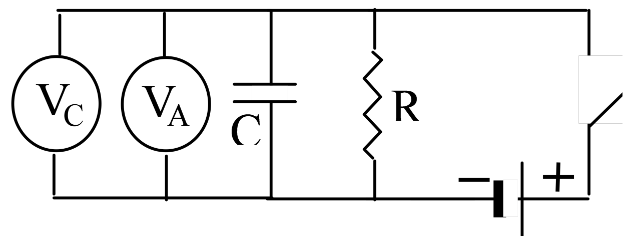
To measure capacitance indirectly, you decide to wire a set of RC circuits and use your computer-based voltage sensor with the Logger *Pro* software to record how the capacitor's voltage decreases over time. You also use the analog meter to visually double check your wiring connections as each decay occurs. In your experiment, you choose to place each of your resistors (10, 15, 22, 33 & 47 ohm) into your circuit one at a time, making a movie and recording voltage data for each RC combination. To confirm that each set-up functions properly, you examine each *QuickTime* movie and watch the analog meter's needle positions throughout the decay process.

1. Preliminary Considerations

(a) To check that you understand the layout of the circuit in this experiment, you decide to draw a circuit diagram. You begin by examining Figure 2 and then proceed to draw this basic RC circuit diagram using the symbols shown below.

> **Hint:** To see the circuit elements more clearly, you can also open the *Quicktime* <RC-10ohm.mov> file and advance to frame one of the movie by using the right arrow key and view the circuit set-up.





(b) To check your understanding of the exponential decay model, examine Equation 2. Which resistor will lead to the fastest decay - a large one like the 47 ohm or a small one like the 10 ohm? Explain the reasons for your answer.

ANS: Intuitively, the resistor serves to reduce the current at any point in time so the rate of charge transfer between plates is slowed down. Mathematically you can see that the greater the R, the greater the t_c (the time constant) and the longer it takes to decay.

2. Activity-Based Findings

You think it's a good idea to analyze the RC-decay plots of the computer voltage data versus the time for each of the five resistors. This analysis will enable you to find the capacitance for each circuit and then to combine these results to get what you hope will be an accurate value for the capacitance.

You have assigned a member of your design team to insert appropriate movies in each of your five Logger *Pro* voltage data files and then synchronize each movie with its voltage data. Your colleague now turns the tables and asks you to perform a curve fit for each RC combination. This is a tough assignment for all of you. It requires a lot of detailed lab work and analysis, but in the end your results show that it is worth the effort.

(a) *Determine the factor B = 1/RC for each data set:* You start the project with the Logger *Pro* file <RC-10ohm.cmbl>. This file is configured so that the <RC-10ohm.mov> *Quicktime* movie is synchronized with the sensor data for the RC-10 ohm circuit. Click on the "Start" button in the Replay window to see a multi-media view of the switch being opened, the voltage being recorded and the *RC*-decay process taking place. Determine the *B-value* for the process by selecting only those voltages that are part of the capacitor decay process and then curve fitting this portion of the data.

> **Hints**: There are several exponential decay functions available in the **Curve Fit menu**, so when choosing the best option be sure to scroll down and use the function that looks the *most* like Equation 2, a *Natural Decay*!
> Also, if you are concerned about units, it can be shown that the unit ohm-farad is equivalent to a second.

Enter your *B value* for the RC-10 ohm combination in the table at the top of the next page.

R (ohms)	B (1/s)	RC (ohm*farads)	C (farads)
10			
15			
22			
33			
47			

Repeat this procedure for each of the other RC combinations.

The next step in this power supply design process is to find the values for the capacitor for each different RC-circuit, the average capacitance value of this capacitor and the standard deviation in this value.

You may use the Logger *Pro* file titled <Basic Table w Calc Column.cmbl> for this purpose. This file allows you to enter each of your 5 values of B for the individual RC Natural Decay fits. It will automatically calculate RC values, each value for the capacitor in the different circuits, the average value of the capacitor and its standard deviation. You may use this option or your calculator or a spreadsheet. It's your option.

(b) Summarize your results is the space below for the average value of the capacitor and its standard deviation.

The average value for C is equal to __________ farads.

The standard deviation for C is equal to __________ farads.

Twice the standard deviation for C is equal to __________ farads.

(c) Which of the suggested options did you use to calculate *C?* Why?

Note to grader: *The reported method will depend on the student's preference. Some may choose to use the pre-packaged LoggerPro file. Others may choose to just choose to use the table in their calculator and solve the problem using their own know how. Any answer is acceptable. It's interesting to learn their preferences.*

3. Reflections on Your Findings

(a) How reliable was your method for finding the value for capacitance? Explain?

ANS The results for the value of the capacitor are 0.090 $\pm$ .002 farads. This means that we know the capacitance to within $\pm$ 0.004 farads or with a $\pm$ 4.4% for 95% of our values. This compares very well to the limitations on the actual values of the resistors that show a $\pm$ 5% variation.

(b) To summarize your work your partner has put all of the exponential decay curves for this series of RC circuit experiments on the graph in Figure 3 below. However, your partner forgot to label the resistances for each curve. The list below indicates the resistors used. One of your last tasks will be to clarify the information on the graph for your design team by matching each curve with its resistor using the labels connected to each curve.

Resistor	**Curve**
15 ohms	*b*
47 ohms	*e*
10 ohms	*a*
33 ohms	*d*
22 ohms	*c*

Explain how you arrived at your answer and why all of the graphs begin at 6 volts.

V_0 in this experiment is 6.0 volts. Therefore, each curve begins with the starting voltage of 6.0 volts. The curves are sequenced from a through e based on their RC-decay constants. The larger the RC-value, the slower the decay process occurs. Since the capacitance of the capacitor is the same throughout all of the experiments, the resistors control the rate of the decay. The a-curve is associated with the smallest resistance, 10 ohms, and the fastest decay. The e-curve is associated with the largest resistance, 47 ohms. The other curves fall sequentially in between these values.

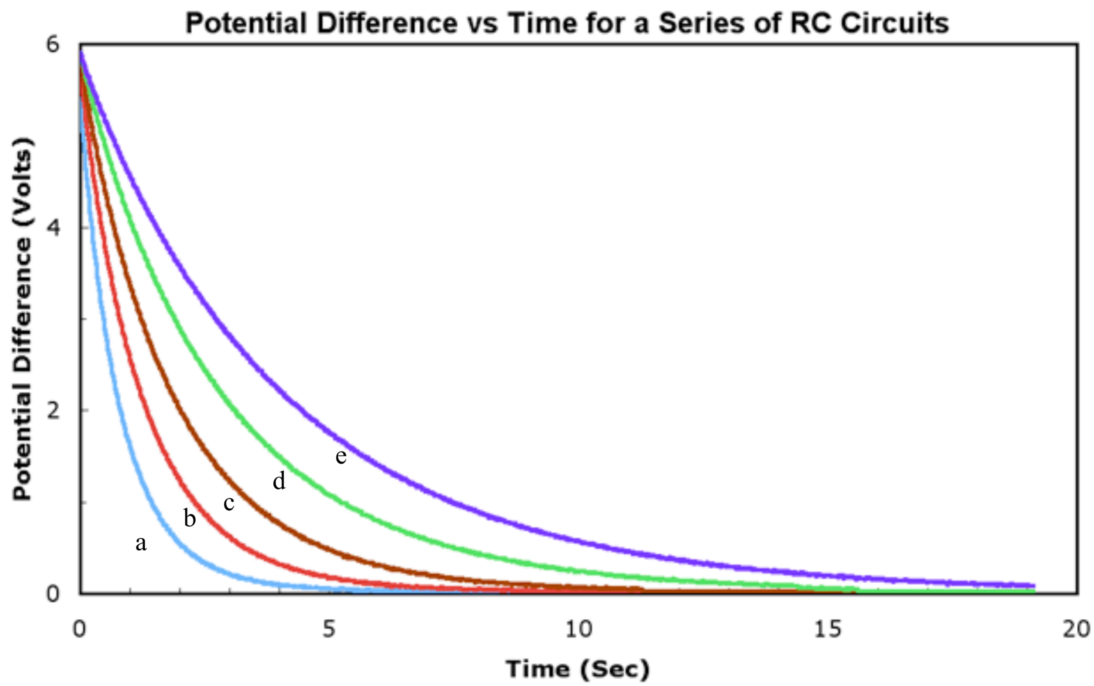


Figure 3: These curves represent the decay data for the corresponding series of resistors used in these circuits with the same capacitor. The curves are labeled alphabetically a through e.

(c) Do you have any ideas how your team could have determined the capacitance of the unknown capacitor differently with the equipment on hand?

ANS Considering that there is no capacitance meter on hand, this was a pretty ingenious way to solve the problem although it is very tedious. Seeing that there is only a 0.005 farad spread between the measured values for C using all five of the circuits, it probably was not necessary to repeat this exercise five times. Three times would have been sufficient to get a reliable average value.