Name:____________________

Date:____________________

# Impulsive Juggling

In this assignment you will be considering the catch–throw process as a slow collision between a juggler's hand and a ball. In particular we would like you to verify the impulse momentum theorem in a typical catch–throw process and the value of the Earth's gravitational acceleration constant.

You'll be tracking the motion of the center ball that the juggler is pointing at in the photo. Each ball has a mass of 105 g.

Completing this task will require you to: (1) carefully obtain *y* vs. *t* data for the center ball for the entire video clip which includes a catch-throw segment and another segment showing the ball's free rise and fall; (2) choose an analytic function that you think ought to fit the free rise and fall data; and (3) perform a curve fit on your data. (4) In addition, you will be exploring the change in the velocity and the acceleration the ball undergoes during the catch throw segment. This should allow you to verify the *Impulse-Momentum Theorem*.

Just for fun, begin your investigation by viewing the video clip entitled <JugglerAll.mov>. Double click on the movie to open it in QuickTime Player. Play the movie or advance it frame-by-frame using the right arrow key (→) on your keyboard. A fun thing to do is play the movie backwards by holding down the left arrow key (←). We found that forward and backward in time for this juggler looks indistinguishable on a computer that is fast enough not to skip frames.

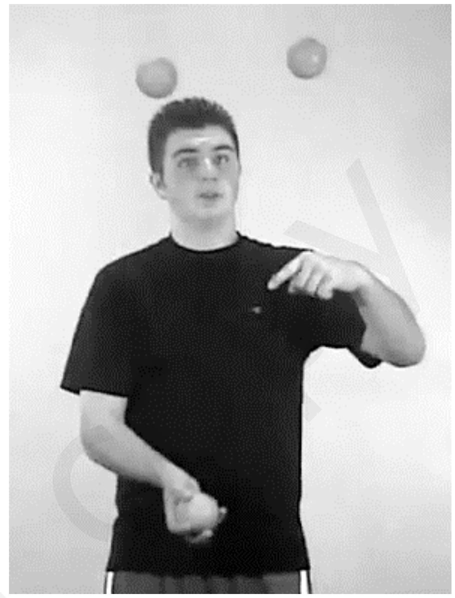


Fig. 1: One of the Nation's top young jugglers, Wes Peden, performs a 3-ball maneuver.

http://www.airplayjugglers.com/

## *1. Preliminary Questions*

**Note:** You will receive FULL CREDIT FOR EACH PREDICTION you make in this preliminary section whether or not it matches conclusions you reach in the next section. As part of the learning process it is important for you to compare predictions with outcomes. DO NOT CHANGE YOUR PREDICTIONS.

(a) Think about a typical toss the juggler is making. What kind of analytic function might be used to fit the graph of *y* vs. *t* while the ball is rising and falling freely in the air? Explain the reasons for your answer. Do you predict a constant acceleration? If so, can you guess its magnitude and direction?

*The y vs. t graph shows a parabolic shape and would be fit with a quadratic equation. When the ball is rising and falling freely it is undergoing a constant downward (vertical) acceleration at the surface of the Earth of* $a_y = -9.8\ \mathrm{m/s^2}$ *due to the force the Earth exerts on objects.*

(b) Think about the very smooth catches and throws the juggler is making. What happens to the velocity? Is it steady or constantly changing?

*The velocity seems to change constantly. When first caught the ball has a downward velocity that slows and passes through zero as the ball reverses direction. It then becomes positive (moving upward) and speeds up as it goes.*

(c) What kind of acceleration do you think the ball experiences as the juggler makes contact with the ball throughout the catch and throw process? Do you think it will be constant or vary? What direction is it, upward or downward. Explain the reasons for your predictions in terms of what you see in the movie.

*The fact that the juggling looks similar when moving forward and back in time and the fact that the motion seems smooth and the velocity seems to change all the time suggests that the acceleration could be constant.*

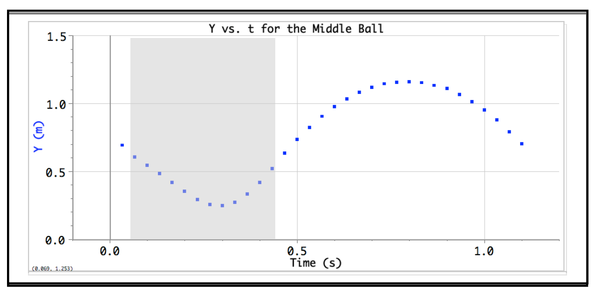
(d) What is the impulse-momentum theorem? In other words, if you know the momentum change during a ball's collision with the juggler's hand, how should the impulse the ball experiences be related to the momentum change? Please define what force should be used to determine the impulse – the gravitational force, the juggler's hand force, or the net force?

*This theorem states that the integral of the net force on an object from some time* $t_1$ to another time $t_2$ *is the same as the change in momentum between these same times. The equation is given by* $\Delta\vec{p} = \vec{p}_2 - \vec{p}_1 = \int_{t_1}^{t_2} \vec{F}^{\,\text{net}}\,dt$

## *2. Activity-Based Questions*

You will be working with a short video clip entitled <JugglerClip4.mov>. It shows Wes' three ball juggling with only two distinct segments on it: (1) a catch-throw segment (frames 3-13); and (2) a free rise and fall (Frames 15-33). We would like you to do a careful curve fit on each of the segments of the video clip with special attention to the catch-throw shown in segment 1.

(a) COLLECT VERTICAL DATA FOR BOTH SEGMENTS: Open the Logger *Pro* file <LPh07-1.cmbl>. The video clip has already been scaled. Skip the title frame (frame 0) by advancing the movie by one frame and then take *y* vs. *t* data in meters for the rest of the frames. First, click on the Add Point tool ( ) and then select the same point **on top of the middle ball** for frames 1 (where $t = 0.033$ s) through 33 (where $t = 1.100$ s). Logger *Pro* will then record the vertical position of the ball as a function of time. Sketch your data points on the graph to the right.




**Note**: It is important that you work very <u>carefully</u>. If you mess up, start over by using the **Clear All Data** feature in the Data Menu.

**Hint**: The grey band corresponds to the catch-throw segment when Wes is touching the ball. We have included a few of key data points as a guide including max and min data.

(b) DO A CURVE FIT FOR THE *y* vs. *t* DATA IN THE FREE FALL SEGMENT: Select the free fall *y* vs. *t* data (frames 15 through 33). Call up the *Curve Fit* feature in the **Analyze** menu and choose the type of equation you think will best match the data (Proportional, Linear, Quadratic, or Cubic, and so on.). Write down the equation that you obtained from your Curve Fit and list the value of the coefficient A with appropriate units. Also report on the time offset after checking the time offset in the lower left corner of the curve fit dialog box. **Note**: Recall that $t_1$ and $t_0$ are alternate notations for the initial time or "Time Offset" for the toss which should be given by $t_0 = 0.500$s.

***ANS****: Equation:* <u>$y = (-4.933\text{ m/s}^2)*(t-t_0)^2 + (2.905\text{ m/s})*(t-t_0) + 0.733$ m</u> for a <u>quadratic fit</u>

A: <u>– 4.933</u> Units: <u>m/s²</u> Time Offset, $t_0$ <u>0.5001</u> Units: <u>s or seconds</u>

**Grader:** *Numbers within 10% of these should receive full credit*

(c) Does the function you chose for your curve fit in segment 2 seem appropriate? In other words, does the fit line match your data well? Yes or no. Explain your basis for deciding whether or not the fit is satisfactory. You may want to look at the reported RMSE.* **Note**: If the curve fit isn't satisfactory perhaps you are trying to use the wrong analytic function and should repeat the analysis in step 2(b) as needed using another function.

***ANS:*** *The RSME value is 0.002 m when the video analysis data are selected extremely carefully. A more casually taken data set gives an RSME of 0.003 m, still quite close. The quadratic fit is just fine.*

**Grader:** *Numbers between 0.001 m and 0.005 m should receive full credit.*

(d) Use the value of A to determine the y-component of acceleration of the ball in the free fall segment. Is it changing or constant? How does if compare to what you predicted in Part 1(a). Cite evidence for your answer. **Hint**: You may use either the $y$ vs. $t$ graph or the $a_y$ vs. $t$ graph to formulate your answer.

***ANS****: According the kinematic equation* $a_y = 2A = \underline{-9.866 \text{ m/s}^2}$ *The acceleration of the ball is constant and quite close to what I predicted. The scale item (the meter stick) may have been in a slightly different plane than the ball toss.*

**Grader:** *Numbers within 10% of this should receive full credit*

(e) INITIAL AND FINAL CATCH–THROW VELOCITIES: Use the data table to determine the instantaneous vertical velocity** of the ball just as it's being caught (frame 3 at $t = 0.100$ s). Also determine the instantaneous vertical velocity of the ball just before its release by the juggler (frame 13 at t = 0.433 s). List the initial and final velocity components with appropriate units and signs using 4-significant figures.

***ANS:*** *Initial:* $v_{1y} = \underline{-1.901 \text{ m/s}}$ *Final:* $v_{2y} = \underline{+3.187 \text{ m/s}}$

**Grader:** *Numbers within 5% of these should receive full credit.*

(g) MOMENTUM CHANGE OF THE CATCH–THROW: Recall that the ball's mass is 105g. Calculate the vertical momentum of the ball when it first falls into the juggler's hand (as in frame 3). Also find the vertical momentum of the ball when it is just about to leave the juggler's hand (as in frame 13). What are the magnitude and direction of the momentum change, $\Delta p_y$, in the vertical direction that the ball undergoes during this time period? Show your calculations. **Beware**: Momentum is a vector quantity! Do not fall into the trap of simply subtracting the magnitude of one quantity from the magnitude of the other.

***ANS:*** *The Vertical Momentum Change between frames 3 & 13 is*

$$p_{1y} = mv_{1y} = 0.105 \text{ kg} (-1.901 \text{ m/s}) = -0.1996 \text{ kg} \cdot \text{m/s}$$

$$p_{2y} = mv_{2y} = 0.105 \text{ kg} (+3.187 \text{ m/s}) = +0.3346 \text{ kg} \cdot \text{m/s}$$

$$\Delta p_y = mv_{2y} - mv_{1y} = +0.3346 \text{ kg} \cdot \text{m/s} - (-0.1996 \text{ kg} \cdot \text{m/s})$$

$$\Delta p_y = +0.534 \text{ kg} \cdot \text{m/s}$$

**Grader:** *Numbers within 5% of these should receive full credit.*

* RMSE stands for Root Mean Square Error. It is a measure of how far away, on average, the data points are from the fitted curve. RMSE is in the units of the y-axis, in this case is meters.

** The numerical derivative we chose here is the weighted average of the slope of 5 points around each point.

(h) IMPULSE OF THE CATCH-THROW: Using the definition of net force as that which causes acceleration, and the fact that $F_y^{net} = ma_y$ and your ability to direct Logger *Pro* to find the integral (as an area under the curve) in the $a_y$ vs. $t$ graph you should be able to determine the impulse between frames 3 and 13. Explain what you did and show your results.

***ANS**: Since the acceleration of any object is directly proportional to the net force on it and inversely proportional to its mass, the impulse the ball experiences is given by its mass (0.105 kg) times the area under the acceleration vs. time graph between frames 3 and 13 (5.026 m/s). I used the integral feature in the analysis menu in* Logger *Pro and selected acceleration data between 0.100 s and 0.433 s and determined the integral to be 5.026 m/s*

$$Impulse = \int_{t_1}^{t_2} \vec{F}^{\,net}\,dt = \int_{t_1}^{t_2} m\vec{a}\ \ dt = (0.105\text{ kg})\ (5.026\text{ m/s}) = 0.528\text{ kg m/s or } 0.528\text{ N s}$$

(i) JUGGLER'S FORCE: If the maximum net force on the ball is 2.85 N, using the information provided can you calculate the maximum vertical force the juggler exerts on the ball during the throw? Begin by drawing a free-body diagram showing the relative magnitudes and directions of the forces on the ball. In your solution be sure to include both an equation for the net forces on the ball and the magnitude and direction of the gravitational force on the ball.

*Gravitational Force is downward and equal to*

$$F_y^{grav} = -mg = -(0.105\text{ kg})(9.8\text{ m/s/s}) = -1.029\text{ N}$$

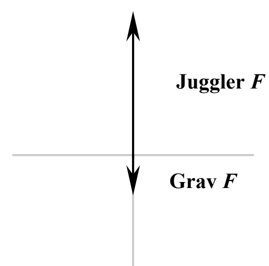
*Juggler Force is upward*

$$F_y^{net} = F_y^{grav} + F_y^{juggler}$$

$$F_y^{juggler} = F_y^{net} - F_y^{grav}$$

$$= 2.85\text{ N} - (-1.029\text{ N}) =$$

$$F_y^{juggler} = +3.88\text{ N}$$



Free body diagram

**Grader:** *Numbers within 5% of these should receive full credit.*

## *3. Reflections on Your Findings*

(a) Does the *Impulse-Momentum Theorem* hold for the catch-throw process? Explain why or why not. **Hint**: In order to say yes, your momentum change and impulse values need to be within at least 10% of each other. (b) What force is relevant in the impulse calculation – the gravitational force, the juggler's hand force, or the net force?

***ANS** (a) According to 2(g) the momentum change is 0.534 kg m/s and the impulse is 0.528 kg m/s or 0.528 N s in part 2(h). These quantities are within 1% of each other. A second more casual analysis gave an agreement to within 7% . (b) The net force is the relevant force in the impulse calculation.*

**Grader:** *Students should receive full credit if they present the comparison and show the values are within 10% of each other.*

(b) How did the acceleration you determined during the catch-throw process compare to what you predicted in question 1(c). Describe what the $a_y$ vs. $t$ graph tells you about the motion of the ball when it is in contact with the juggler's hand?

***ANS**: I expected the acceleration to be constant and larger than g by the amount needed to turn it around. It seems to take time for the juggler to make full contact with the ball and to let it go. The positive acceleration builds up to a maximum during the catch phase and diminishes during the throw phase.*

**Grader:** *Students should discuss the $a_y$ vs. t graph and use it to describe the motion of the ball when it is in the juggler's hand. Also the student should compare $a_y$ with g to receive full credit.*