Name:______________________
Date:______________________

2D Vectors: Pool Ball Displacement, Velocity and Speed

A careful analysis of two-dimensional motion is more complex than it is for motion in one dimension – especially when calculating velocities and speeds. In this activity you are going to analyze a movie entitled <Pool_Ball_Bounce.mov> that shows the 2D motion of a pool ball rolling on a level pool table. The ball starts with its center at point A, moves "up" and to the left to point B, bounces off the edge of the table and rolls "down" and to the left so that its center reaches point C. You can view the pool ball's motion in *QuickTime Player* by double clicking on the file <Pool_Ball_Bounce.mov>.

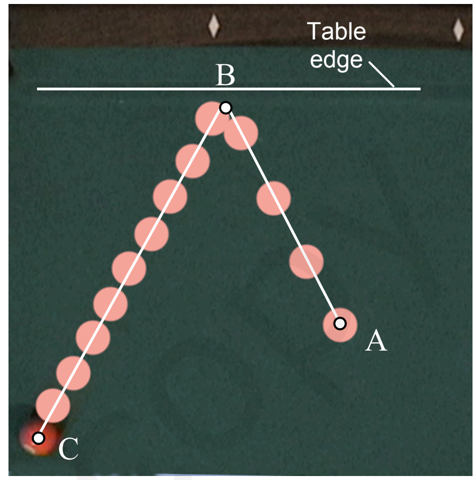

Fig. 1: A pool ball travels from point A to B to C as it bounces off a table edge.

The purpose of this activity is to use the Logger *Pro* video analysis tools to: (1) scale a movie; (2) use displacement vectors to find average speed and average velocity; and (3) show that the magnitude of the average velocity, $\left|\langle \vec{v}_{A \to C} \rangle\right|$, of object moving in 2D is not always the same as its average speed, $\langle S_{A \to C} \rangle$.

Are Speed and the Magnitude of Velocity the Same? "The person on the street" often uses the terms speed and velocity as if they are the same thing. By now you are probably familiar with the fact that in physics speed is a scalar quantity that has no direction associated with it while velocity is a vector quantity that tells what direction an object is moving. Since the magnitude of a vector is a scalar, can't we just be a little more careful and say that the terms *speed* and *magnitude of velocity* are the same? The answer is not always.

Average speed is the total distance something moves divided by time interval the object took to travel that distance. For example if you take 5.0 seconds to walk in a circle having a circumference of 10 meters and arrive back at the same place your average speed it 2.0 m/s. One the other hand *average velocity* is a vector quantity given by the displacement of an object divided by time interval the object took to undergo that displacement. But, a displacement just depends on where the object was when you started measuring a time interval and where it is at the end of the time interval. So if you walk in a complete circle your displacement is zero and your average velocity and its magnitude are both zero even through you walked a total distance of 10 meters!

In the activities that follow you will be applying ideas about speed and velocity to the pool ball bounce.

1. Preliminary Questions

(a) *Speed of the pool ball:* Is the ball moving faster before it collides with the edge of the pool table or after? What is your evidence?

ANSWER: The ball is moving faster before it collides with the edge of the pool table. The evidence is that when advancing the Quicktime movie on a frame-by-frame basis, the ball moves a greater distance between frames before it collides with the edge of the table than afterward. Since the time interval between frames remains constant the ball is moving more slowly after hitting the edge of the table..

(b) Based on your viewing the movie and the definitions of average speed and average velocity do you expect that the average speed of the pool ball to be:

- ☐ greater than the magnitude of the average velocity
- ☐ less than the magnitude of the average velocity
- ☐ the same as the magnitude of the average velocity

(c) Explain the reason(s) for your answer.

ANSWER: Looking at figure 1 or the movie itself, it is obvious that the displacement vector which points from point A to point C is a lot shorter than the total distance (length of the path) the ball travels. Since the time of travel is the same in both calculations, the magnitude of the average velocity is going to be considerably smaller than the average speed.

2. Activity Based Questions

Scale the Movie and Locate the Ball's Position: In order to answer the questions that follow you'll need to use the Logger *Pro*. Double click on file <LPh04-1.cmbl> to open a video analysis file with <Pool_Ball_Bounce.mov> inserted. We suggest you scale the movie using the distance between the pool table markers shown on the first frame and go ahead and find the location of the center of the ball in each of the frames.

(a) Find the Average Speed of the Ball in cm/s to two significant figures and explain how you determined the speed. Also, show any calculations.

Hint: Consider using the photo distance tool ( ) in the last frame of the analyzed movie.

ANSWER: In the last frame of the movie I can see the marks where the center of the ball was in each frame. I used the photo distance tool to estimate that the distance from A to B is about 42 cm and the distance from B to C is about 63 cm. The ball's total path length is the sum of these two distances. The time elapsed between the first frame (Fr#0) and the last frame (Fr#13) is 0.867 s.

So the average speed is $\langle S_{AC} \rangle = (42\text{ cm} + 63\text{ cm})/0.867\text{ s} = 1.2\text{x}10^2\text{ cm} \pm 10\%$

(b) Find the mathematical expression for vector that describes the displacement of the ball when it has traveled from A to C. Describe how you determined the vector and show your calculations. *Reminder*: Displacement is a vector and should be written with its $\hat{\text{i}}$ and $\hat{\text{j}}$ components.

ANSWER: I can use the data table to determine the x and y coordinates relative to the origin (at the bottom left in the movie frames) of the ball in frame 0 (Point A) and the ball in frame 15 (Point B). The difference between these two locations is the displacement vector we are looking for and is given by

$$\Delta\vec{r}_{A\to C} = \vec{r}_{C} - \vec{r}_{A} = (x_{C} - x_{A})\hat{\text{i}} + (y_{C} - y_{A})\hat{\text{j}}$$

$$\Delta\vec{r}_{A\to C} = (11.4\text{cm} - 61.3\text{cm})\hat{\text{i}} + (5.5\text{cm} - 25.1\text{cm})\hat{\text{j}}$$

$$\Delta\vec{r}_{A\to C} = (-49.9\text{cm})\hat{\text{i}} + (-19.6\text{cm})\hat{\text{j}}$$

Note*: There might be up to a 10% variation in the values of the reported components.*

(c) What is the average velocity of the ball as it travels from point A to point C? Explain how you determined it and report your result to two significant figures. *Reminder*: The fundamental definition of average velocity is the displacement divided by the total time.

ANSWER: According to the data table. The time elapsed between the first frame (Fr#0) and the last frame (Fr#13) is 0.867 s. So

$$\langle\vec{v}_{A\to C}\rangle = \frac{\Delta\vec{r}_{A\to C}}{\Delta t}$$

$$\langle\vec{v}_{A\to C}\rangle = \frac{\Delta x_{A\to C}}{\Delta t}\hat{\text{i}} + \frac{\Delta y_{A\to C}}{\Delta t}\hat{\text{j}} = \frac{-49.9\text{cm}}{.867\text{s}}\hat{\text{i}} + \frac{-19.6\text{cm}}{.867\text{s}}$$

$$\langle\vec{v}_{A\to C}\rangle = = \frac{-49.9\text{cm}}{.867\text{s}}\hat{\text{i}} + \frac{-19.6\text{cm}}{.867\text{s}}\hat{\text{j}} = (-58\text{cm/s})\hat{\text{i}} + (-23\text{cm/s})\hat{\text{j}}$$

Note*: There might be up to a 10% variation in the values of the reported components.*

(e) What is the magnitude of the average velocity of the ball as it travels from A to C? Show your calculations.

ANSWER:

$$\left|\langle\vec{v}_{A\to C}\rangle\right| = \left|\frac{\Delta\vec{r}_{A\to C}}{\Delta t}\right| = \sqrt{\left(\frac{\Delta\vec{x}_{A\to C}}{\Delta t}\right)^2 + \left(\frac{\Delta\vec{y}_{A\to C}}{\Delta t}\right)^2}$$

$$\left|\langle\vec{v}_{A\to C}\rangle\right| = \sqrt{\left(-58\frac{\text{cm}}{\text{s}}\right)^2 + \left(-23\frac{\text{cm}}{\text{s}}\right)^2}$$

$$\left|\langle\vec{v}_{A\to C}\rangle\right| = 62\ \frac{\text{cm}}{\text{s}}$$

Note*: There might be up to a 10% variation in the value of the reported result.*

3. Reflections on Your Findings

(a) How well did your prediction in Part 1(b) for the relative magnitudes of average velocity and average speed compare with the calculations you just made in Part 2? Explain

GRADER: Any honest attempt gets full credit

(b) You should have found that the average speed of the ball is much greater than the magnitude of its average velocity. Explain why.

The average speed of the ball, 120 cm/s, is much greater than the magnitude of its average velocity, 62 cm/s*, because the fundamental definition for speed is the* ***distance covered*** *divided by the time it takes to cover that distance and the average velocity is the* ***displacement*** *divided by the time elapsed. The calculation of the average speed in this problem illustrates that the path of the pool ball is very important. The ball covers a much greater distance during the time elapsed as compared with its actual displacement during that same time period. Therefore, the average speed would be expected to be greater than the average velocity.*

(b) Can you think of way that the pool ball can move from point A to point C such that its average speed and the magnitude of its average velocity are the same?

ANSWER: If the pool ball moves directly from point A to point C without bouncing off the edge of the table then the length of its displacement vector and the length of its path and the time it takes to go from A to C would be exactly the same. In this case $\left|\langle\vec{v}_{A\rightarrow C}\rangle\right| = \langle s_{A\rightarrow C}\rangle$

(c) This exercise has required many calculations for the motion of the pool ball. Reflect on what you have learned from completing this detailed analysis of the motion of the pool ball and summarize your conclusion below.

ANSWER: This is an open-ended question. Students should reflect on the difference between average speed and average velocity. They should also comment on the fact that the velocity-magnitude and speed of the object do not require vector notation but that velocity does require vector notation and is usually given by components. This example also shows the student that the value for the average speed and the magnitude of the average velocity are quite different and helps them understand why this is so. Students may also comment on how the exercise has clarified how to use raw data to perform the calculations.