Physics Experiment:
Center-Loaded Rectangular Beam Displacement
Dependence on Load, Length, Height, and Base

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Introduction

The heart of experimental physics involves determining power relationships between variables related to specific physical phenomena. The electrical power generated from flowing water spinning a turbine is proportional to the cube of the volume of water moved per unit time. For a fixed mass of gas under a constant temperature, absolute pressure and volume are inversely proportional. The intensity of light emitted from a point source is inversely proportional to the square of the distance from the source. The magnetic field measured along the axis of a small neodymium magnet is inversely proportional to the cube of the distance from the magnet. These are all examples of power relationships that are commonly discovered by students in the physics laboratory.

With the release of the Next Generation Science Standards (NGSS), the National Science Teachers Association (NSTA) has been promoting these standards from elementary through college levels. These standards have strong implications for the teaching of both science and engineering. On the engineering front, the Vernier Structures and Materials Tester (VSMT) has been designed as a platform to easily test the strength of bridges, structures such as trusses, and beams. Vernier has developed three experiment files with activities supporting engineering practices: individual structure analysis, class bridge competition, and evaluating the displacement of a beam supported at both ends. The latter file explores the equation for a center-loaded beam that is supported at both ends.

$$\Delta = \frac{FL^3}{48EI}$$

The beam's elastic displacement at mid-span is Δ, *F* is the load, *L* is the span length, *E* is the modulus of elasticity, and *I* is the area moment of inertia. If we consider a solid rectangular beam of length *L*, base *b*, and height *h*, then the area moment of inertia is $bh^3/12$, and the equation becomes the following.

$$\Delta = \frac{FL^3}{4Ebh^3}$$

This equation suggests a perfect opportunity for the physics student to investigate power relationships experimentally. Students could, without knowledge of the equation, design experiments to determine that elastic displacement is directly proportional to the load, directly proportional to the cube of the beam's length, inversely proportional to the beam's base, and inversely proportional to the cube of the beam's height. The physics student would be involved in NGSS activities, including asking questions and defining problems, developing and using models, planning and carrying out investigations, analyzing and interpreting data, using mathematics and computational thinking, constructing explanations and designing solutions, engaging in argument from evidence, and obtaining, evaluating, and communicating information.

General Considerations

There are a number of things that are useful to keep in mind when doing the beam experiments discussed here.

1. The investigations here are for solid, rectangular beams; not I-beams, trusses, or other complicated structures.
2. Wood strips can be obtained from most hobby stores, either locally or online, such as Michaels or Hobby Lobby.
3. Basswood is much preferred over balsa wood. Balsa wood tends to be too soft and brittle, breaking quite easily under relatively small forces.
4. You don't need to have thick pieces of wood. All experiments can be done with wood that is ¼″ thick, or less. Doing so will also allow you to keep the load forces smaller so that less energy is released in the event of beam failure.
5. Although not required, a caliper is useful for more accurate measurement of beam thickness.
6. When doing the experiments, there is no need to apply forces that will bend the beams to the breaking point.
7. The experiments described here make use of a U-bolt and quick link to securely connect the beams to the VSMT.
8. As per safety recommendations in the *VSMT User Manual*:
    - Safety glasses should be used to protect eyes from flying debris in the event that a beam fails.
    - The threaded parts on the U-bolts should be attached so that a sufficient amount of the threaded component is engaged.
    - Quick links should be secured and not left open.
9. When doing the experiments described here, there is no need to know tackle mass. The reason for this is that when investigating the relationship between a beam's length, base, or height versus elastic displacement, the force simply needs to be constant. The exact value of the force is not critical. The suggested analyses of the raw VSMT data will provide for constant force.
10. The VSMT Force Sensor should be attached to an analog port and the VSMT Displacement Sensor should be attached to a digital port.
11. It is suggested that your students begin each of the experiments with a new LabQuest qmbl or Logger *Pro* cmbl file. The VSMT Displacement Sensor does not auto-ID when selecting a new file, so your students will have to manually set up the VSMT Displacement Sensor. See page 3 of the *VSMT User Manual* for additional steps on how to do this.
12. Make sure that both the VSMT Force Sensor and VSMT Displacement Sensor are zeroed with the tackle ever so slightly slack before starting data collection.
13. Students should have force *vs*. time and displacement *vs*. time graphs visible.
14. Different runs for the same experiment file should be used to vary the independent variable. It is helpful to do the first run with the independent variable value that causes the beam to bend most easily, but not allowing it to bend to the breaking point.

Subsequent runs with other values for the independent variable then need to have load forces not much larger than that for the first run. For example, when investigating the effect of beam length on elastic displacement, run 1 could be for a length of 40 cm, run 2 for a length of 32 cm, run 3 for a length of 24 cm, run 4 for a length of 16 cm, and run 5 for a length of 8 cm.

Investigating the Relationship between Elastic Displacement and Force

The left side of Figure 1 shows typical graphs of force *vs*. time and displacement *vs*. time. These graphs are obtained during data collection while turning the displacement wheel on the VMST. Plateaus simply represent time intervals when the displacement wheel is at rest. In order to determine the power relationship between displacement and force, the student needs to construct a graph of displacement *vs*. force, and then find a model that fits the graph. A typical graph of displacement *vs*. force is shown on the right side of Figure 1. When a linear fit is applied, a correlation of 0.9995 clearly shows that elastic displacement is directly proportional to the load force applied at the center of the beam.

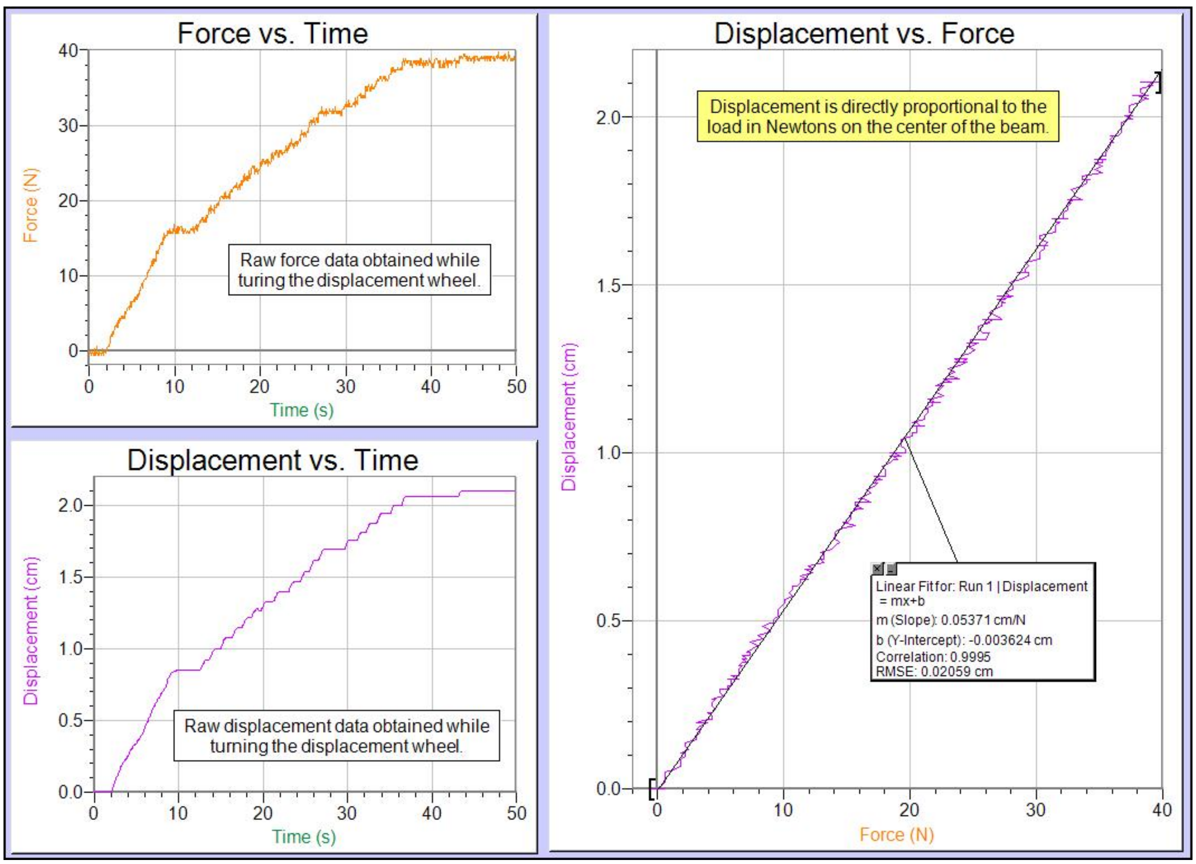

Figure 1

Investigating the Relationship between Elastic Displacement and Beam Length

Basswood beams with base = ½″ and height = ¼″ worked well for investigating the relationship between beam length and elastic displacement. Figure 2 shows such a beam on a VSMT with the length set to 40 cm (20 cm + 20 cm) on the crossbars. The beam was secured using a U-bolt and quick link. Five experiment runs were done with beam lengths of 40, 32, 24, 16 and

8 cm. The 40 cm run was done first, as it represents the beam that feels the least stiff when trying to bend it by hand. Subsequent runs on stiffer beams then never really needed to exceed the maximum load force that was used for the 40 cm beam.

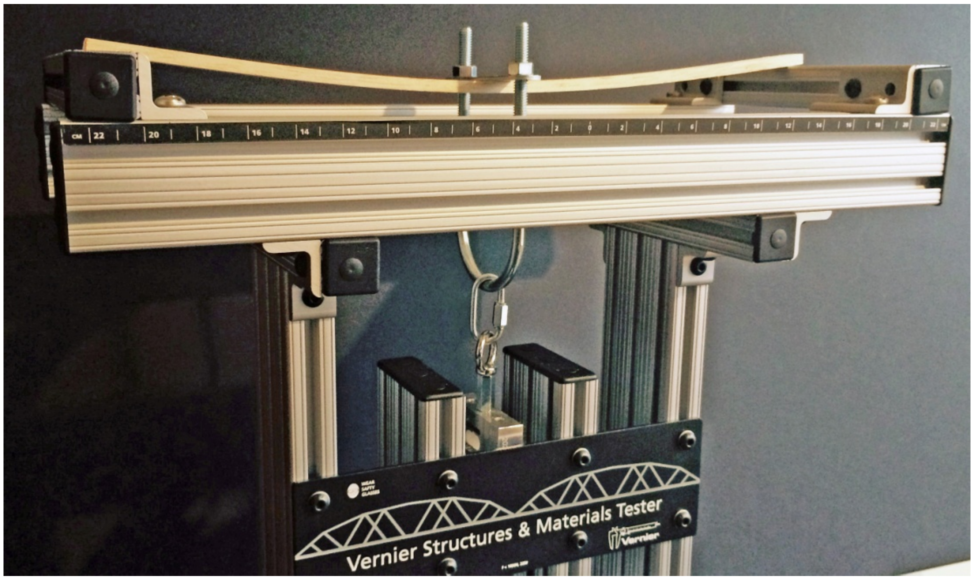

Figure 2

Figure 3 shows a graph of elastic displacement *vs*. force for the beams of varying length. Creating this graph was a four-step process in Logger *Pro*:

1. Plot all runs on the same displacement *vs*. force graph.
2. Use Analyze/Linear Fit and select all of the runs.
3. Use Linear Fit Options to hide the Helper objects by selecting the Show in Graph checkbox to clear it. The straight lines of best fit will still remain on the graph.
4. Use Analyze/Interpolate to obtain displacement values for a specific force near the upper force range for the longest beam. In Figure 3 a force of 38.10 N was used as shown by the vertical line on the graph. The corresponding beam displacements are also shown in the Helper object.

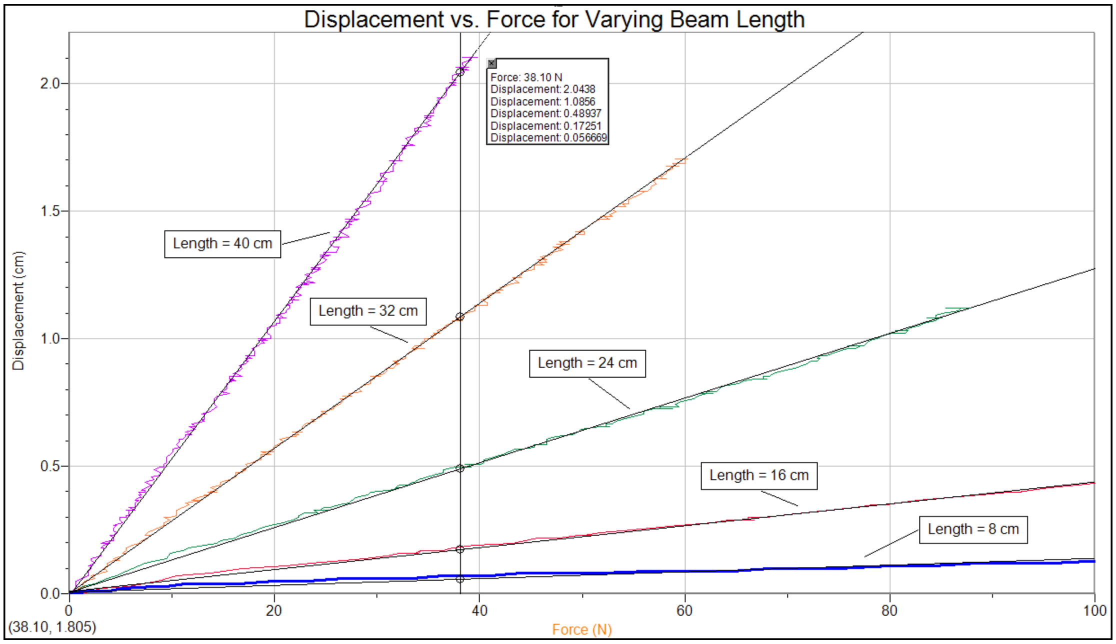

Figure 3

The beam lengths and corresponding elastic displacement data from Figure 3 were then manually entered into a new Logger *Pro* cmbl file. A graph of beam displacement *vs*. length was set up as shown in Figure 4. The red dots show the five experimental data points and they immediately suggest some kind of a power relationship between elastic displacement and beam length. Using Analyze/Curve Fit/Variable Power to model the data, powers of 2, 3, and 4 were tried. The model for power 3 clearly follows the data the best of all. We have convincing evidence that beam displacement is directly proportional to the cube of the beam's length.

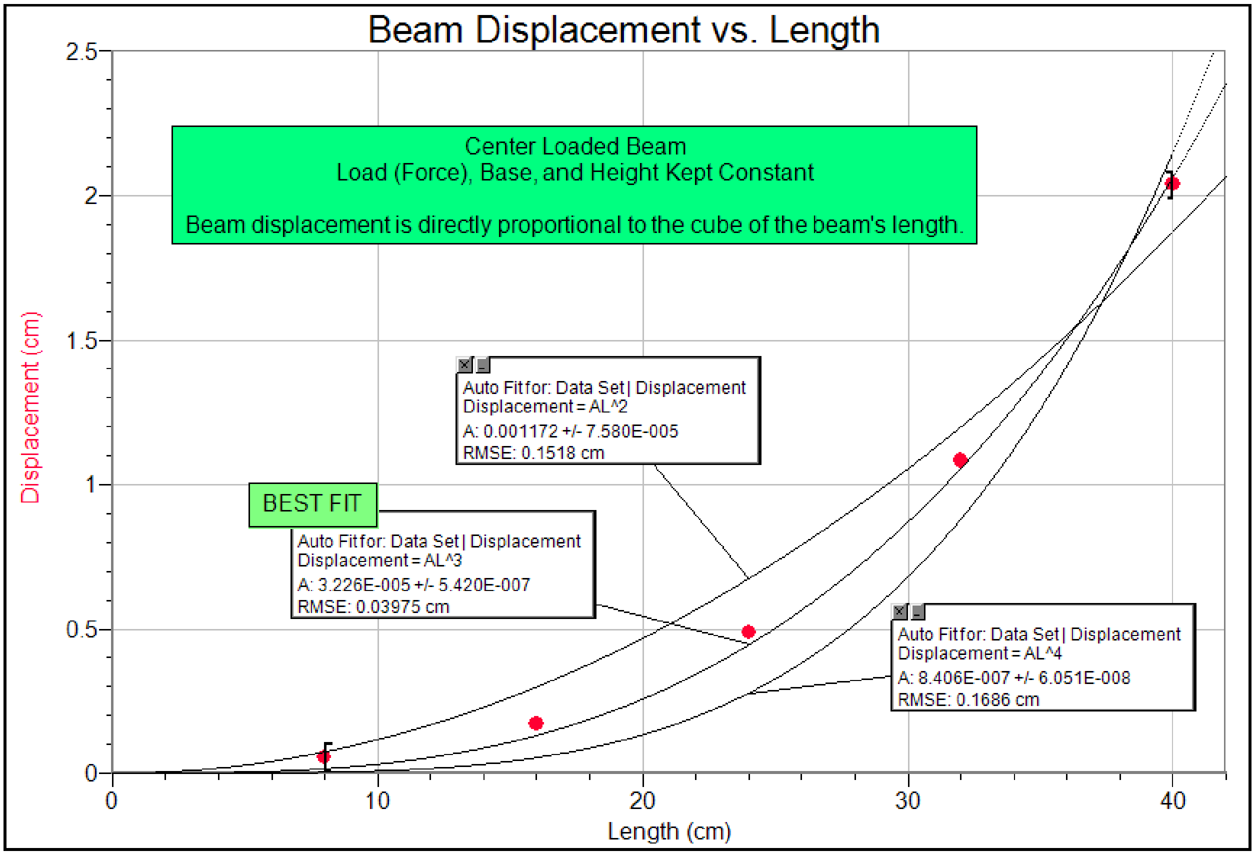

Figure 4

Investigating the Relationship between Elastic Displacement and the Beam's Base

As shown in Figure 5, basswood beams of length 6″ and thickness 1/16″ proved to work well for investigating the relationship between elastic displacement and the beam's base. Bases of 1 cm, 2 cm, 3 cm, 4 cm, and 5 cm were used.

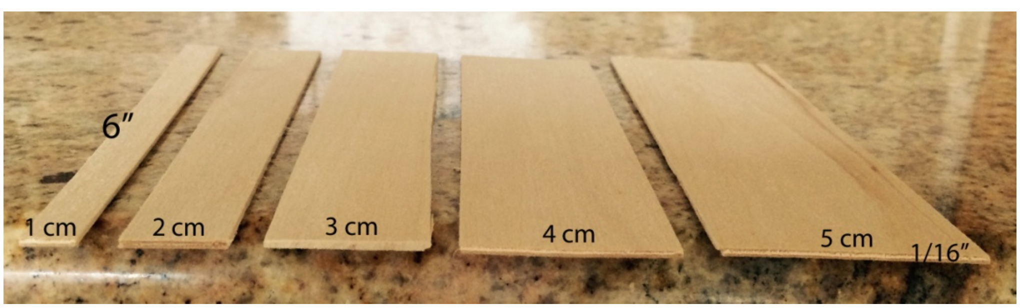

Figure 5

Figure 6 shows a graph of elastic displacement *vs.* force for the beams of varying base. This graph was created using the same four step process that was described in the previous section. In Figure 6, a force of 12.02 N was used for analysis and is shown by the vertical line on the graph. The corresponding beam displacements are shown in the balloon.

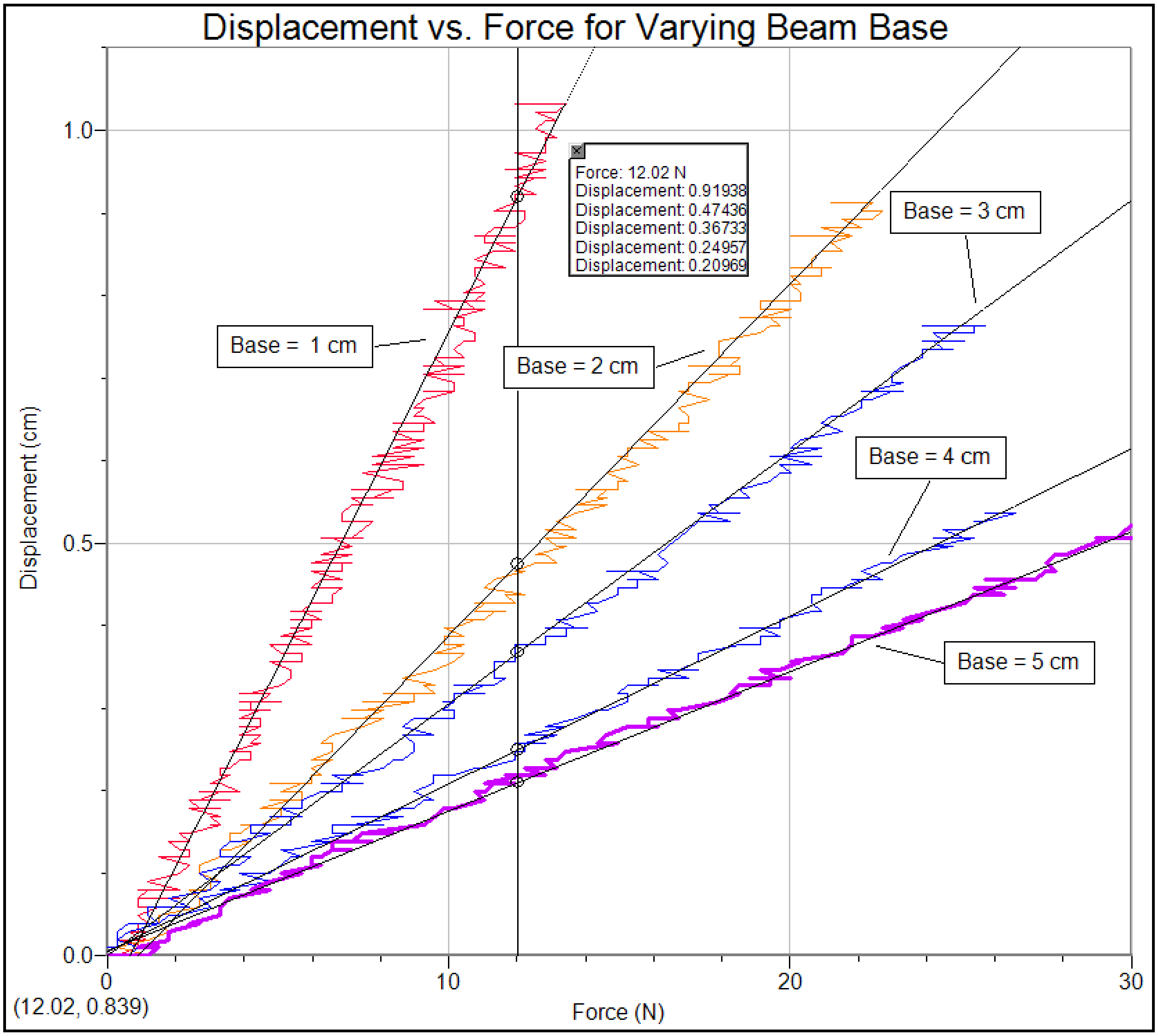

Figure 6

The beam bases and corresponding elastic displacement data from Figure 6 were then manually entered into a new Logger *Pro* cmbl file. A graph of beam displacement *vs*. base was then set up as shown in Figure 7. The red dots show the five experimental data points and they immediately suggest some kind of an inverse power relationship between elastic displacement and beam base. Using Analyze/Curve Fit/Variable Power to model the data, powers of -0.5, -1, and -1.5 were tried. The model for power -1 clearly follows the data the best of all. We have convincing evidence that beam displacement is inversely proportional to the beam's base.

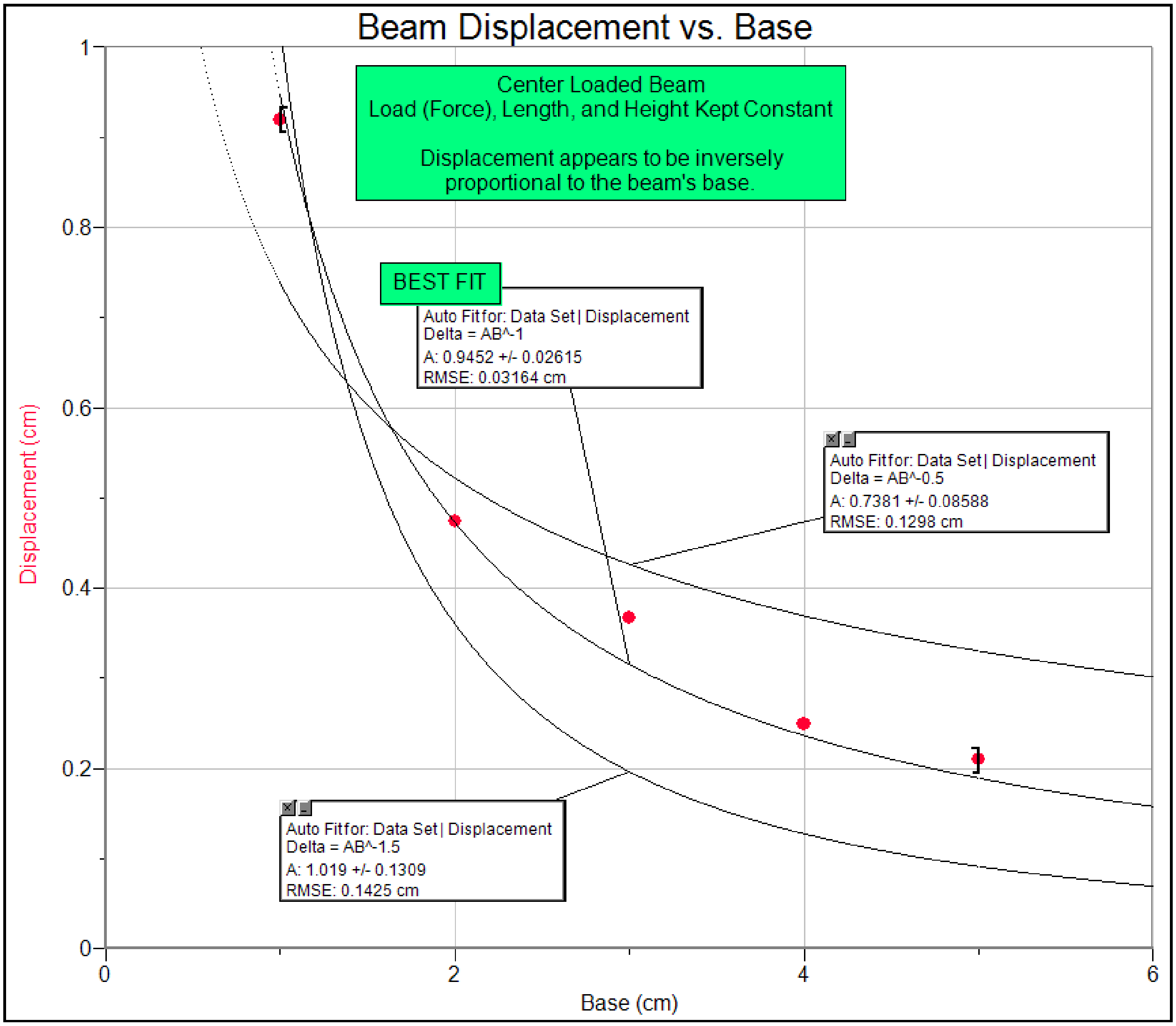

Figure 7

Investigating the Relationship between Elastic Displacement and the Beam's Height

As shown in Figure 8, basswood beams of length 12″ and base 2″ proved to work well for investigating the relationship between elastic displacement and the beam's height. Heights of 1/16″ (0.169 cm), 3/32″ (0.236 cm), 1/8″ (0.327 cm), and 1/4" (0.642 cm) were used. Metric measurements, shown in parentheses, were obtained using a caliper. The basswood thicknesses are standard thicknesses found in many hobby shops.

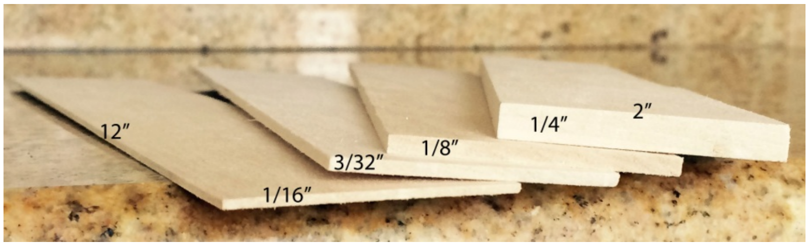

Figure 8

Figure 9 shows a graph of elastic displacement *vs*. force for the beams of varying height. This graph was created using the same four steps that was described earlier. In Figure 9, a force of 17.05 N was used for analysis and is shown by the vertical line on the graph. The corresponding beam displacements are shown in the Helper object.

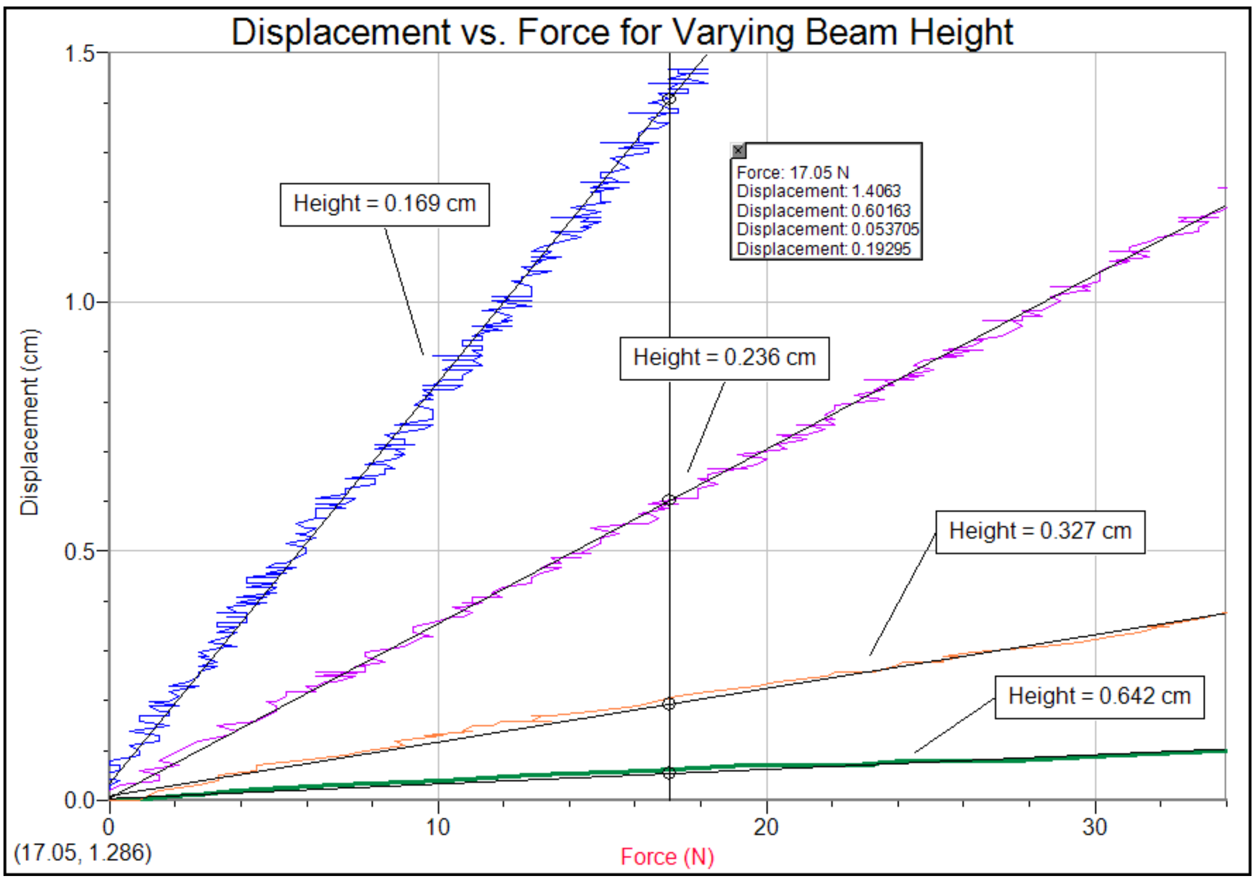

Figure 9

The beam bases and corresponding elastic displacement data from Figure 9 were then manually entered into a new Logger *Pro cmbl* file. A graph of displacement *vs*. height was then set up as shown in Figure 10. The red dots show the four experimental data points and they immediately suggest some kind of an inverse power relationship between elastic displacement and beam height. Using Analyze/Curve Fit/Variable Power to model the data, powers of -2, -3, and -4 were tried. The model for the power -3 follows the data the best of all. We have convincing evidence that beam displacement is inversely proportional to the cube of beam's height.

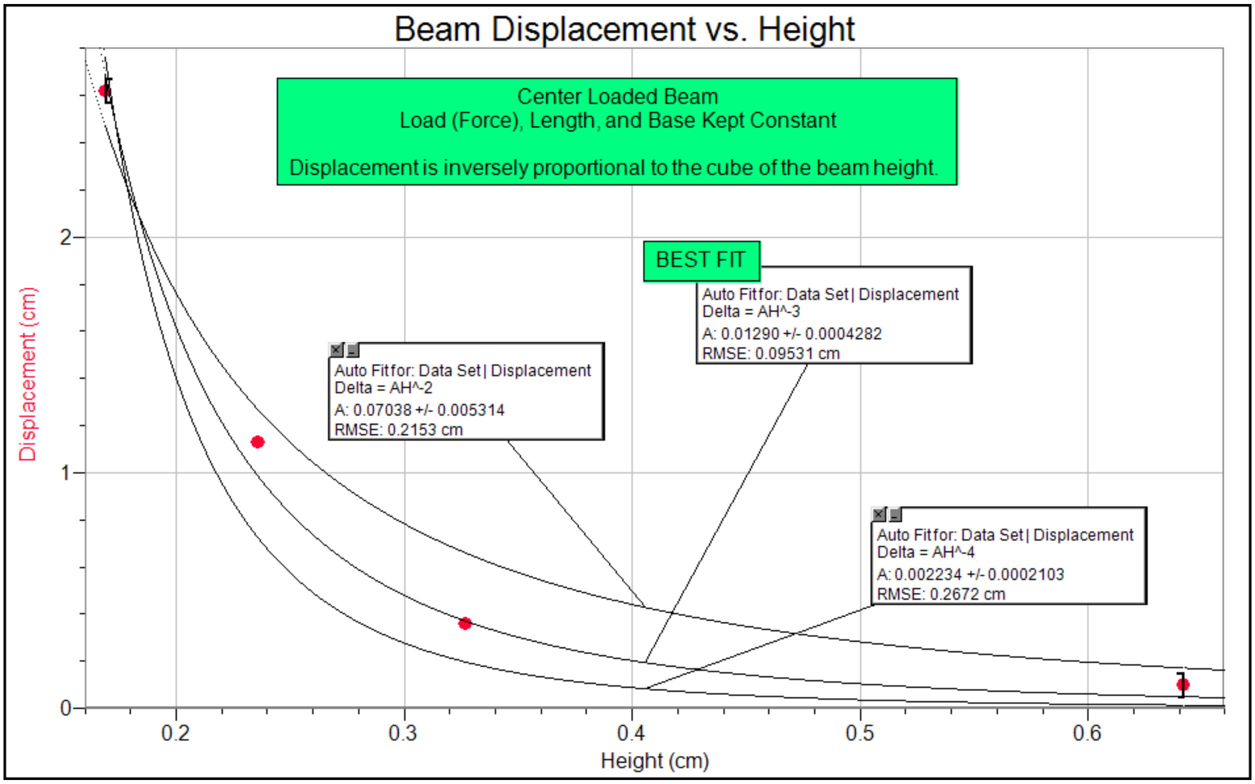

Figure 10

Additional Investigations

1. Hobby Lobby sells a package of square dowels that are 12″ long and having length of sides varying from 1/8″ to 1/2". Based upon what you have learned in the previous experiments, hypothesize on the relationship between elastic displacement and the length of a side of the square dowels. Carry out an investigation with the VSMT to test your hypothesis.
   Teacher explanation: For a square dowel, the base and height are the same. Since elastic displacement is inversely proportional to bh^3, then elastic displacement for a square dowel with side s would be inversely proportional to s^4. This is a great investigation. How many phenomenon have your students investigated that have an inverse fourth power relationship?
2. It was stated in the introduction that the area moment of inertia *I* of a solid, rectangular beam of height *h*, and base *b* is $bh^3/12$. Derivation of area moments of inertia is commonly accomplished via calculus. Do some Internet research to learn more about area moment of inertia and learn how to derive the formula $I = bh^3/12$ for a solid, rectangular beam.
3. Many hobby shops sell round dowel rods of varying diameters. Hypothesize the relationship between diameter and elastic displacement. Carry out an investigation with the VSMT to test your hypothesis.
4. Do some Internet research to find an equation for the area moment of inertia for a round beam. Does this provide further evidence to confirm your hypothesis made in the previous step?