Projectile Motion

Introduction

Up to this point it is likely that you have examined the motion of an object in one dimension only—either falling vertically under the influence of Earth's gravity or on a horizontal or inclined surface.

In this experiment, you will examine the behavior of a projectile—an object moving in space due to some initial launching force. Such an object can undergo motion in two dimensions simultaneously. Using the Vernier Video Analysis app, you will compare features of the position *vs*. time and velocity *vs*. time graphs with those of one-dimensional motion.

Objectives

In this experiment, you will

- Use video analysis techniques to obtain position, velocity, and time data for a projectile.
- Analyze the position *vs*. time and velocity *vs*. time graphs for both the horizontal and vertical components of the projectile's motion.
- Create and analyze your own video of an object undergoing projectile motion.

Materials

Vernier Video Analysis app in a web browser on a computer, Chromebook, **or** mobile device
camera capable of recording digital video (e.g., the camera on a phone or tablet)
tripod or other equipment to support the device used to record the video
meter stick or some other object to provide scale
projectile: a brightly-colored ball works well
video editing software (optional)

Pre-Lab Investigation

Your instructor will launch a projectile. Observe its motion carefully, then discuss its position *vs*. time and velocity *vs*. time behavior. Sketch a graph of the projectile's horizontal velocity *vs*. time, and sketch another graph of vertical velocity *vs*. time. Share your sketched graphs with your group or with the class.

Sample

Investigation

Part 1 Analysis of an existing video

1. Launch Vernier Video Analysis. Import the movie "Basketball Shot." Play the movie once so you can be familiar with the motion.
2. Add sufficient points to the basketball shot to show the basketball's motion graphically.
3. Apply a scale to the video and decide where to put the origin of the axes.
4. Adjust your graph to display only the *x* position data.
5. Do Part 1 of the Analysis, and then return to continue with Part 2 of the investigation.

Part 2 Production and analysis of your own video

For this part, you need a device capable of recording video such as a digital camera or smartphone; it can be the same device you used in Part 1 for Vernier Video Analysis. Keep the following tips in mind when you shoot your video.

- It is best to have a plain background that provides sufficient contrast with the object you are investigating. Good lighting is essential; recording outside on a cloudy day will provide better light than recording inside.
- Set up the device on a tripod so that it is pointed squarely at the background, and so that the plane of motion is perpendicular to the view.
- Position the camera as far from the plane of motion as is practical in order to reduce problems with scaling and parallax. Use the zoom feature to fill the screen with the motion.
- The object used for scaling must be in the same plane as the motion of the projectile (see Figure 1).

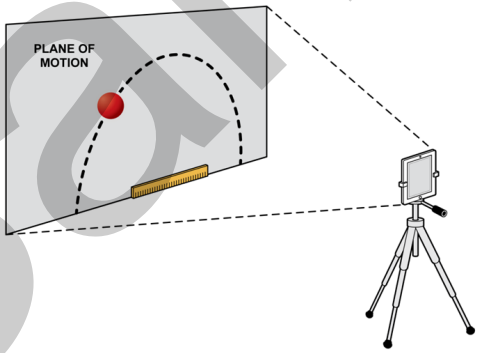

Figure 1

- Once you have captured your video, transfer the video to the device you will use for the analysis, if necessary. Optional: If you have captured more video than you need, trim the video to contain just the motion you plan to analyze.

6. Capture video of your object undergoing projectile motion, and import your movie into Vernier Video Analysis.

Sample

7. Add sufficient points to the basketball shot to show the projectile's motion graphically. Then, apply a scale to the video and decide where to put the origin of the axes.

8. Adjust your graph to display only the *x* position data, and complete the analysis.

Analysis

Part 1 Analysis of an existing video

1. Examine the graph of *x* position *vs*. time. If it appears to be linear, fit a straight line to your data. If the slope of the graph appears to change abruptly, select each segment, one at a time, and fit separate straight lines to each portion of the graph that appears to be linear.
   a. Click-and-drag or touch-and-drag across the graph to select the portion of the graph that appears linear.
   b. Click or tap Graph Tools, , choose Apply Curve Fit, and apply a linear curve fit.
   c. Repeat Steps a and b for additional linear regions of the graph, if desired.

2. Write the equation that describes the *x* position *vs*. time behavior of the ball in each segment; be sure to include units.

3. Based on what you have learned in previous experiments, write a description of the horizontal component of the motion of the projectile. Note when any change in the horizontal component of the motion occurs.

4. Now, examine the graph of *y* position (Y) *vs*. time.
   a. Click or tap the vertical axis label to open the Plot Manager. Turn on the *y* position and turn off *x* position.
   b. Fit an appropriate curve to this graph (or to each portion of the graph). To do this, click or tap Graph Tools, , choose Apply Curve Fit. Select the curve fit model that best fits your data, and apply the curve fit.
   c. Write the equation that describes the *y* position *vs*. time behavior of the ball in the first segment; be sure to include units.

5. Based on what you have learned in previous experiments, write a description of the vertical component of the position of the projectile.

6. Now, to test your analysis in Step 5, examine the graph of *y* velocity *vs*. time.
   a. Click or tap the vertical axis label to open the Plot Manager. Turn on *y* velocity (Y Velocity) and turn off *y* position.
   b. Click-and-drag or touch-and-drag across the graph to select the portion of the graph that appears linear.
   c. Click or tap Graph Tools, , choose Apply Curve Fit, and apply a linear curve fit. Fit a straight line to the first portion of the graph only, if there are multiple linear regions.
   d. Repeat Steps b and c for additional regions of the graph, if desired.

7. What can you say about the rate of change of the *y* velocity as a function of time? How does the value of the slope of the linear fit compare to the acceleration of a freely falling object? Summarize in a sentence or two.

Sample

8. Compare the a and b parameters (including values and units) of the curve fits you performed in Step 4 to the slope and intercept of the linear fit you performed in Step 6.

9. Explain the differences in the horizontal and vertical components of the velocity of the projectile in terms of the force(s) acting on the projectile after it was launched.

10. Display the velocity vectors for the motion of the projectile, and then explain how the vectors correspond to the motion of the projectile in the vertical and horizontal directions. To display the velocity vectors, follow these steps:
    a. Use View, ⊞, to display the video if it is hidden.
    b. Click or tap Vectors, ↗, to add a vector display. (Click or tap Add, ⌖, if the Vectors option is not visible.)
    c. Enable the velocity vectors only. To do this, click or tap the visibility toggle next to Velocity until Components, ⮆, is displayed.
    d. Adjust the Scale Factor and Vector Frequency so you can clearly see the vectors.
    e. Explain how the displayed vectors correspond to the motion of the projectile in the vertical and horizontal directions that you have also described with graphs and equations.

Part 2 Production and analysis of your own video

11. Perform the analysis of your movie as you did with the video provided to you in Part 1.

Extensions

1. Suppose that, in the shooting of your video, you placed the meter stick used for scaling against the wall you used for your background. However, the plane of the ball's motion was 0.50 m in front of the wall. The distance between the camera and the wall was 5.0 m. Would this error result in a value for the acceleration of gravity in your analysis of the *y* velocity *vs*. time graph that was smaller or larger than the accepted value? By what factor would this value differ from the expected value? Explain using a diagram.

2. Repeat the production and video analysis of a projectile, but this time use an extended body (i.e., an object that cannot be readily modeled by a point-particle). Consider carefully how best to mark the position of such an object during its motion. Interpret your position *vs*. time and velocity *vs*. time graphs as you did for Part 1.

3. Using either the analysis of the basketball shot sample video or the analysis of the video you created, model the vertical motion based on ideal conditions. Create a new calculated column and use the initial upward velocity of the object and the acceleration of gravity to calculate the ideal vertical velocity. Use the graph to compare the video-based velocities with the model-based velocities. Suggest reasons for any discrepancies.

   Note: Instructions for creating a calculated column are found in the Vernier Video Analysis User Manual, **http://www2.vernier.com/manuals/video-analysis-manual.pdf**

Sample

Projectile Motion

Projectile motion is an ideal candidate for analysis using video. This experiment should be performed only after students have had the opportunity to explore the behavior of an object moving with constant velocity as well as one that undergoes uniform acceleration.

Note that Part 2 of this activity requires students to create their own video. There are suggestions in the instructions for capturing video that are useful for analysis. Additional tips for collecting, editing, and analyzing video are available in Appendices A, B, and C.

If you need to alter this activity for time, you may omit Part 2 (in which students create and analyze their own video) of this activity by editing it out of the word-processing file of the instructions.

If this is your students' first time using Vernier Video Analysis, we recommend giving them the Guided version of the instructions. If your students are already familiar with using Vernier Video Analysis, consider giving them the Abridged version of the instructions.

Objectives

In this experiment, the student objectives include

- Use video analysis techniques to obtain position, velocity, and time data for a projectile.
- Analyze the position *vs*. time and velocity *vs*. time graphs for both the horizontal and vertical components of the projectile's motion.
- Create and analyze your own video of an object undergoing projectile motion.

During this experiment, you will help the students

- Recognize that the horizontal component of a projectile's motion is best described as an object moving with constant velocity.
- Recognize that the vertical component of a projectile's motion is best described as an object undergoing uniform acceleration.
- Recognize that, neglecting air resistance, the acceleration of the object very nearly equals a_g.
- Capture and edit a video for analysis in the Vernier Video Analysis app.

Equipment Tips

Students have the opportunity to create and analyze their own projectile video in this activity. To do this, students will need either a phone or digital camera, a tripod or other supporting device, and some object they can use for scaling. For most students, the first object they should use for a projectile should be a small ball that is massive enough not to experience significant slowing from air resistance. A ball that bounces will provide an opportunity to examine how the horizontal component of the motion changes after the ball bounces. Students are likely to want to examine the behavior of more "interesting" objects. While more advanced students may be able to handle this

Sample

well, encourage most students to save those objects for the Extension activity and remind them to be safe.

Pre-Lab Discussion

This experiment should be performed only after students have had the opportunity to explore the behavior of an object moving with constant velocity as well as one that undergoes uniform acceleration. Demonstrate tossing a ball, then ask the students to describe the position-time behavior and velocity-time behavior of the ball. They are likely to describe the ball's path as parabolic but may have more difficulty describing how its velocity changes with time. Suggest that separate analyses of the horizontal and vertical components of the ball's motion could be revealing. Ask students to predict and sketch *x* velocity *vs*. time and *y* velocity *vs*. time graphs for the projectile.

Lab Performance Notes

We recommend that students first learn how to do video analysis on the provided "Basketball Shot" movie. Analysis of this movie is likely to yield results that are easier to interpret as other movies may suffer from poor scaling, perspective problems, or blurred images.

The basketball in the provided movie bounces on the pavement, and has a different horizontal velocity before and after the bounce. Consider whether to instruct students to graph all the motion or only the motion prior to the bounce. Edit the student instructions accordingly.

Sample Results and Analysis

Steps 1–3

Depending on how you instruct students to do this activity, they may have either one or two linear segments of the graph. In the sample graph Figure 1, the first slope is from data collected before the basketball hit the ground and the second slope is from after the basketball hit the ground (showing that the ball slowed down).

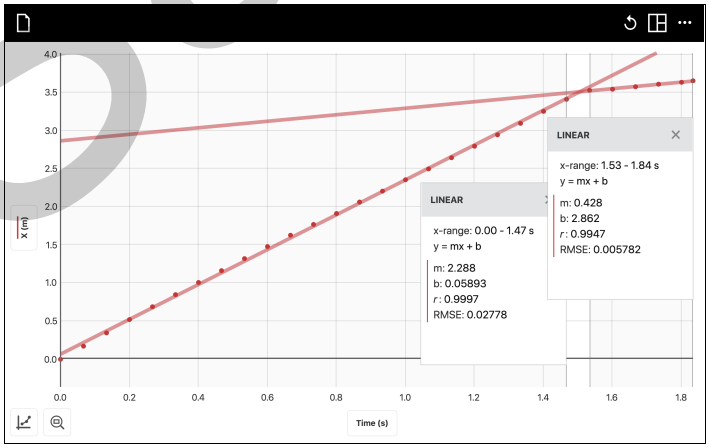

Figure 1 Graph of x *position* vs. *time with two linear curve fits*

Sample

The equations for the graph in Figure 1 are $x = (2.288 \text{ m/s})t + 0.05893 \text{ m}$ and $x = (0.428 \text{ m/s})t + 2.862 \text{ m}$.

Steps 4–5

Students should recognize that a quadratic fit is appropriate for the *y* position *vs*. time graph. They may perform a quadratic fit to just the motion before the ball bounces (see Figure 2), or they may add the quadratic fit for each parabolic segment.

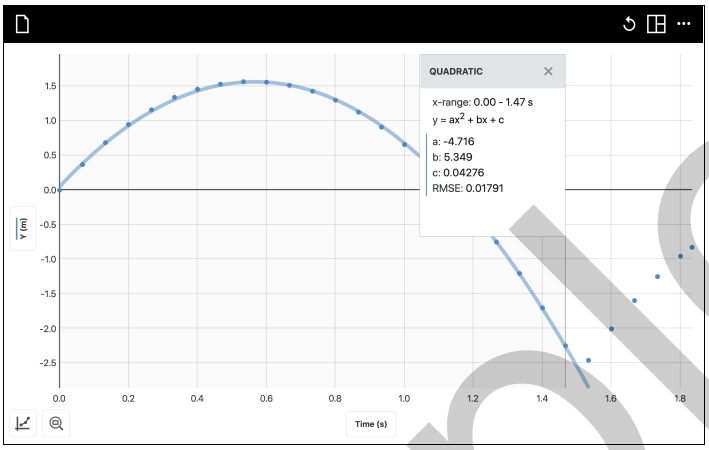

Figure 2 Graph of y *position* vs. *time with one quadratic curve fit*

The equation for the first segment is $y = (-4.716 \text{ m/s}^2)t^2 + (5.349 \text{ m/s})t + 0.04276 \text{ m}$. The fact that a quadratic fits the *y* position *vs*. time graph so nicely indicates that the ball is undergoing acceleration.

The *a* coefficient for the quadratic is close to the expected value. However, it is obviously lower than the expected value. It is useful to have a class discussion about why the value is lower than expected. You may bring up the difficulty of scaling to the meter stick, since the ends are difficult to discern in the video. In addition, the basketball may not be moving in the same plane as the meter stick (the meter stick may be farther from the camera). Finally, depending on the lens of the camera, there is a certain amount of distortion in the video image the farther away you get from the center of the frame. The apparent location of the ball may be affected by this distortion. Errors such as these can be reduced by doing the following:

- Use a longer scale object with good contrast against the background and ensure that it is in the plane of the motion.
- Ensure that the camera is "square" to the plane of the motion and aimed at the center of the motion.
- Change the camera lens or the lens setting to a "narrow focus" or telephoto.
- Move the camera farther away from the motion.

Sample

Steps 6–7

The interval used for performing the linear fit should not include the first few and last few points.[1] The *y* velocity *vs*. time graph shows that the vertical component of the ball's velocity changed at a constant rate (see Figure 3). Students should recognize the rate of change of the velocity as the acceleration of the ball. If the object used for scaling and the plane of the ball's path coincide, the value of the acceleration should be nearly –9.8 m/s^2, a_g.

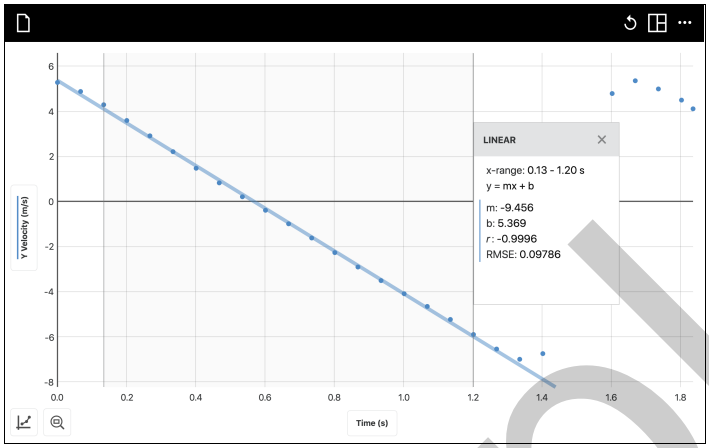

Figure 3 Graph of y *velocity* vs. *time*

Step 8

By comparing the value of a in the quadratic fit of the *y* position *vs*. time graph to the slope of the linear fit of the *y* velocity *vs*. time graph, students should be able to conclude that a is half of the acceleration. Furthermore, note that for the parabola before the bounce, the b parameter is the same as the intercept of the line of best fit to the corresponding *y* velocity *vs*. time graph. This intercept is the *y* velocity for the first frame used in the analysis. Once these connections have been made, students should recognize that the general equation for the position of an object moving with constant acceleration, $y = \frac{1}{2}at^2 + v_0 t + y_0$, describes the vertical component of the motion of the projectile.

Step 9

From their understanding of the relationship between motion and forces, students should conclude that no net force acts on the projectile in the horizontal direction, except when it makes contact with the ground. Note that the *x* velocity decreases after the bounce. The projectile accelerates in the vertical direction because of the force exerted on it by the earth. The sign of the acceleration is consistent with the direction of gravitational force.

Step 10

Students add horizontal and vertical velocity vector components to the video view. Note that the horizontal velocity vectors show that the *x* velocity is constant (but with different values before and

[1]This is due to the way that Vernier Video Analysis calculates the velocity of the object at each clock reading. See **www.vernier.com/til/1011** or the Instructor Notes for Activity 2, Accelerated Motion in this book.

Sample

after the basketball bounces). The vertical velocity vectors are consistent with an object that is undergoing acceleration in the (negative) *y* direction.

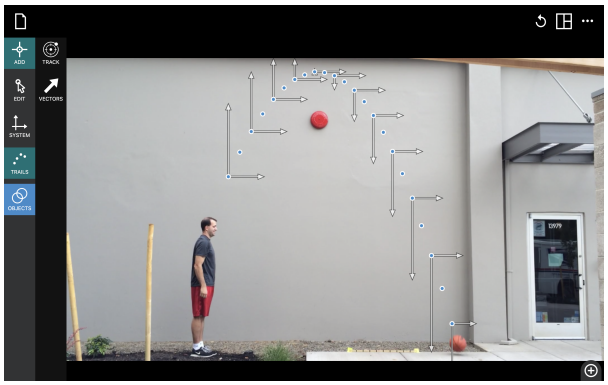

Figure 4 Vectors representing the velocity components of the projectile

Step 11

Student results will vary depending on the student video. Results should be similar to those found in Part 1.

Extensions

1. If the object used for scaling were farther from the camera than the plane of the motion, the video analysis will report values of position and velocity that are larger than they actually are. This will result in a value of the slope of the *y* velocity *vs*. time graph that is 5.0/4.5 = 1.1 times larger than the accepted value (a_g would appear to be –10.9 m/s^2).

2. For the analysis of the motion of an extended body that undergoes rotation during the motion, students should mark the center of gravity (a piece of contrasting tape works well) before recording the video so they can mark the position of its center of mass. Analysis of the motion of the center of mass of the extended body should yield results much like those of a point particle.

3. Modeling the ideal motion can be done using the expression A*X+B found in the calculated column expressions. A is the acceleration of gravity, –9.8 m/s^2, and B is the initial upward velocity. For the purpose of this activity, the student may use either the initial velocity from the Y Velocity column or the *y* intercept of the linear fit for the *y* velocity *vs*. time graph (see Figure 5). If some students or groups use the Y Velocity column and others use the *y* intercept, students can discuss the differences between the models and the reasons for them.
Note: Instructions for creating a calculated column are found in the Vernier Video Analysis User Manual, **http://www2.vernier.com/manuals/video-analysis-manual.pdf**

Sample

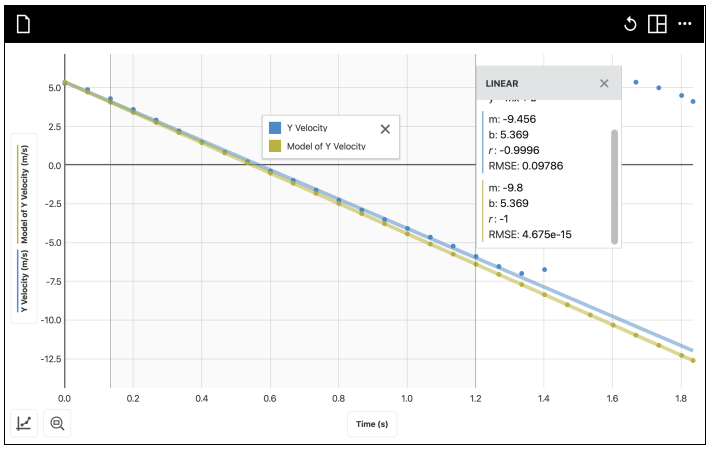

Figure 5 Two linear models, one based on the original curve fit (blue) and one based on the y *intercept of the original curve fit (light green), are displayed.*

Sample