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## Inverse Variation in Pressure Data

Imagine squeezing a small balloon filled with air. The more you squeeze the balloon the smaller the balloon gets; as you relax your grip the balloon expands. In the example of the balloon, the squeezing of your hand represents pressure and the size of the balloon represents volume. This experiment will investigate the mathematical relationship between pressure and volume known as Boyle's Law. Since the volume of a balloon is difficult to measure, you will use a syringe with cubic centimeter graduations to identify volume. (Note: $1 \mathrm{cc}=1 \mathrm{~mL}$ )


## OBJECTIVES

- Collect and graph pressure versus volume data for a sample of air.
- Investigate the mathematical relationship between pressure and volume.


## MATERIALS

TI-84 Plus or TI-84 Plus Silver Edition graphing calculator
Vernier EasyData application

## Vernier Easy Link

Vernier Gas Pressure Sensor with included syringe

## PROCEDURE

1. Turn on your TI-84 Plus or TI-84 Plus Silver Edition graphing calculator and make sure that it is on the home screen. Plug the Vernier Gas Pressure Sensor into EasyLink. Now plug your EásyLink" into the USB port of the graphing calculator. The EasyData application will automatically start and the Main screen will be displayed.
2. With the syringe disconnected from the Gas Pressure Sensor, move the piston of the syringe until the leading edge of the inside black ring is positioned at the 10.0 cc mark.

3. Attach the syringe to the Gas Pressure Sensor with a gentle half turn.
4. Set up the data collection.
a. Select File from the Main screen, and then select New.
b. Select Setup from the Main screen.
c. Select Events with Entry
5. You are now ready to collect pressure and volume data. It is best for one person to handle the syringe and for another to operate the calculator.
a. Select Start to begin data collection.
b. Move the piston so the front edge of the inside black ring is positioned at the 5 cc line on the syringe. Hold the piston firmly in this position until the pressure value displayed on the calculator screen stabilizes.
c. Select Keep. (The person holding the syringe can relax after Keep is selected.) Type in 5.8, the gas volume (in cc), on the calculator. (The additional 0.8 cc is the amount of air inside the Gas Pressure Sensor. Including this value greatly improves your results.) Select OK to store this volume-pressure data pair.
d. To collect another data pair, move the piston to 7.0 cc . When the pressure reading stabilizes, select Keep and enter the total volume, remembering to add the 0.8 cc correction to the syringe volume.
e. Continue with this procedure using volumes of $10.0,12.0,15.0,17.0$, and 20.0 cc .
f. Select Stop when you have finished collecting data.
6. A graph of pressure $v s$. volume will be displayed. Use (D) to examine the data points along the curve. As you move the cursor, the volume ( $x$ ) and pressure ( $y$ ) values of each data point are displayed above the graph.

7. Select $\sqrt{\text { Main }}$ to return to the Main screen. Select $\widetilde{\text { Quit }}$ from the Main screen. Select $\widetilde{\text { OK }}$ to exit the EasyData application.
8. Disconnect EasyLink from the USB port of your calculator.

## ANALYSIS

1. Redisplay the graph outside of EasyData.
a. Press 2 nd [STAT PLOT].
b. Press ENTER to select Plot1 and press Enter again to select On.
c. Press zoom and select 9:ZoomStat .
d. Press trace and use the cursor keys to review the coordinates of the data points.
2. You might have noticed that in order to make the volume of the syringe less than 10 cc , you needed to push the plunger in. The smaller the volume the more you had to push. What do you think would have happened to the pressure readings if you had made the volume less than 5 cc ? How much pressure would you have to exert on the syringe to make the volume zero? What implications does this have for the $y$-intercept of your graph?
3. Suppose the syringe you used could be adjusted to $50 \mathrm{cc}, 100 \mathrm{cc}$ and beyond. What do you think would have happened to the pressure readings as you increased the volume to values greater than 20 cc ? Would the pressure readings ever reach zero or below? Explain. What implications does this have for the $x$-intercept of your graph?
4. The rate the pressure changes is not the same for all volumes. This is evident by the shape of the graph. What shape would the graph have had if the rate of change of pressure were everywhere the same? Based on your data, when is the rate of change in pressure the greatest? When is it the least? Explain how you know.
5. The answers to questions 2-4 help narrow down the possible choices of functions that can be used to model the data. One mathematical model that supports the answers is the rational function $y=\frac{k}{x}$. Plot this function with your data.
a. Press $r=$ and enter the following into Y1: K/X
b. On the home screen, type 500 STOD $K$ and press ENTER.
c. Press GRAPH to view a graph of the function $y=500 / x$.
d. Adjust the value of K until you are satisfied that the function matches your pressure $v s$. volume data.
e. What value of K best matches your data? Are you comfortable that this function is a good model for your data? Explain.
6. Two quantities vary inversely, if the product of the quantities is constant, for any given ordered pair. Determine if your data supports the idea that pressure and volume vary inversely.
a. Press star and select 1:Edit
b. In list L 3 , calculate the product of volume and pressure for each ordered pair ( $\mathrm{L}_{1} \mathrm{~L}_{2}$ ).
c. Are the values in L3 roughly constant?
d. How does the average of list L3 compare to the value you found for K in question 5?
7. How is the function $y=\frac{k}{x}$ related to the idea of inverse variation discussed in question 6 ?

## Teacher Tips

1. The data collection for this exercise is generally easy. Boyle's Law data is excellent for showing variables that are inversely related. Nevertheless, there is an assumption that the air temperature in the syringe is constant. In this experiment you are ignoring the fact that compressing a gas causes it to warm up thus affecting the results slightly.
2. The maximum pressure the Gas Pressure Sensor will measure can be exceeded in this experiment. To get best results, do not attempt to measure pressure when the volume is less than 4 cc .
3. An interesting way to show the inverse nature of the pressure-volume data is to show the direct (linear) relationship between the pressure and the reciprocal of the volume data.
4. Another way to model the data would be to perform a power regression. The power regression model is $y=a \cdot x^{b}$. If pressure and volume data for a fixed quantity of gas are really inversely proportional, you would expect $b=-1$ and $a=\mathrm{K}$.
5. Challenge your students to develop a method for calculating the internal volume of air in the sensor. They would need to collected data without adjusting for the internal volume then model the data using the equation $y=\frac{k}{x+d}$, where $d$ represents the internal volume.
