Activity 7

# Water Flow from a Tank: Parabolic Trajectory

## Introduction

Bernoulli's principle is an energy conservation statement for fluid flow. For a steady flow, the sum total of all forms of energy—kinetic, potential, and internal—at each point along the flow must be the same. This means that when the fluid is flowing faster (greater kinetic energy) at one point than another, there must be a decrease in potential (e.g., gravitational) or internal energy.

Torricelli's theorem is a special case of Bernoulli's principle, relating the outflow velocity from a water tank to the pressure head of the fluid within the tank. The greater the pressure head (depth of the liquid), the faster the fluid flow.

## Objectives

In this experiment, you will

- Use video analysis techniques and projectile motion relationships to obtain velocity data for a stream of water.
- Use Bernoulli's principle to derive Torricelli's theorem.
- Predict the exit velocity of the stream of water and compare it to the measured velocity.

## Materials

Vernier Video Analysis app in a web browser on a computer, Chromebook, or mobile device

"Water-Tank" video file

## Pre-lab Investigation

1. Launch Vernier Video Analysis and import the "Water-Tank" movie. Play the movie once or twice to observe the change in the trajectory of the water stream as the height of the column of water decreases.

2. Think about a water molecule at each of these locations:
   - the top of the column of water (in the tank)
   - the exit hole
   - the end of the stream (bottom of the frame)

   Identify the energy transitions from the top of the water column, to the exit hole, and to the bottom of the video frame.

3. What form of energy is associated with the fluid pressure at the exit hole?

4. Share and discuss your predictions with classmates.

# Procedure

1. Make the movie window large enough to easily see the water level in the tank. There are two ways to do this: 1) Click or tap the handle on the divider between the video and the other elements on the screen, and drag the divider to the right, or 2) use View, ⊞, to remove the graph and data table from view.

2. Set the origin and the scale in the video.
   a. Click or tap Reset Video, ⏮, to quickly return to the first frame of the video.
   b. Click or tap System, ⊥. You will see new icons appear for Scale and Origin. Scale, 1.0, is already selected.
   c. Move the centers of the scale circles to align with the 30 cm (0.3 m) distance measurement in the first frame.
   d. Adjust the displayed length to 0.30 m.
   e. Click or tap Origin, ⊙. Place the origin at the exit hole of the water stream.

3. Mark the location of the water level in the tank. By default, each time you mark the object's location, the movie advances by one frame.
   a. Click or tap Trails, ⋰, to hide the points after you have marked them. (This will make it easier to mark your points.)
   b. Click or tap Add, ⌖.
   c. Decide where on the surface of the water you will mark its location (i.e., the center, the right edge where the water meets the wall of the tank, or other). It is important to be consistent when marking the location; always place the crosshairs on the same location on the water level.
   d. Position the crosshairs at the chosen location on the water surface, and then click or tap to add the first point. **Note**: If you are using a phone or tablet, once you place the crosshairs you can tap anywhere in the video frame.
   e. Continue adding points until you have marked every frame that shows the tank. (The last two frames show an up-close image of the hole in the tank.)

4. Mark the location of the position of the stream of water where it falls along the bottom of the video frame.
   a. Click or tap Reset Video, ⏮, to quickly return to the beginning of the video.
   b. Click or tap Objects, ⌬, then select **+ADD NEW OBJECT**.
   c. Click or tap Add, ⌖, and mark the position of the stream of water along the bottom of the video frame (see Figure 1). Mark every frame that shows the stream of water.
   d. Click or tap Objects, ⌬, then click or tap Object 1 options, ⋯. Change the name to Level. Click or tap **<Back** to save the new name.
   e. Click or tap Object 2 options, ⋯. Change the name to Stream. Click or tap **<Back**.

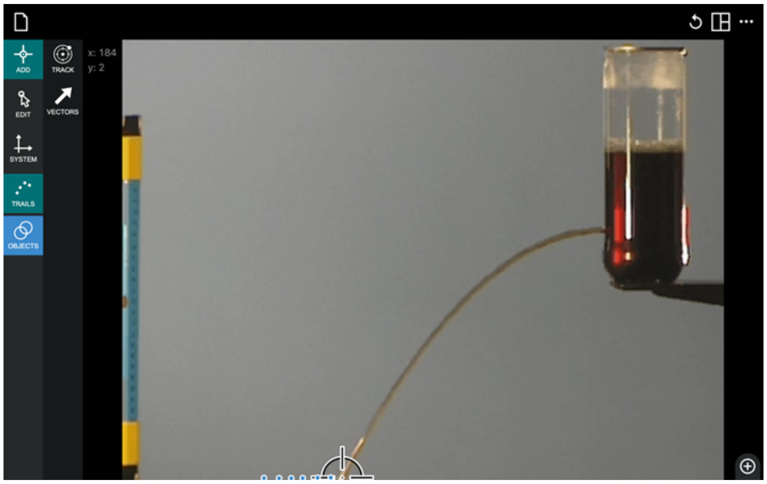


*Figure 1  Collecting data for the water stream along the bottom edge of the video*

5. Review your marked points. Should you wish to edit a point, click or tap Edit, . This allows you to move or delete a mismarked point.

6. Use View, , to display the graph and data table, and hide the video.

7. Vernier Video Analysis defaults to display both the *x* and *y* positions of marked objects as a function of time. For this experiment, we are interested in the *y* component of the position of the Level and the *x* and *y* components of the position of the Stream. To change which data are displayed, click or tap the vertical axis label to open the Plot Manager. Turn off X Level and verify that Y Level, X Stream, and Y Stream are turned on.

8. Save your Video Analysis file.

# Analysis

1. Consider the stream of water exiting the container. It exits the container horizontally and then follows the trajectory of a projectile. How far vertically does the water fall from the exit hole to reach to the bottom edge of the video frame? Use the Y Stream data to find this value.

2. Using the standard kinematics equation $y = \frac{1}{2}gt^2 + v_{y0}t + y_0$, determine the amount of time it takes for water exiting the container through the hole to reach the bottom edge of the video frame.

3. Like a projectile fired horizontally from a cliff, the water stream's exit velocity from the hole is purely horizontal. That means the water's horizontal (X) position when it reaches the bottom edge of the video frame is equal to its exit velocity multiplied by the time it took to fall the vertical distance. Use this relationship to create a calculated column for the exit velocity for the water.
   a. Write an expression for the exit velocity of the water in terms of the horizontal landing position of the stream (X Stream) and the time it takes the water to fall, which you calculated in Step 2.

b. In the data table, click or tap Column Options, ⋯, for the X Stream column and choose Add Calculated Column.
c. Enter **Exit Velocity** as the name of the column and **m/s** as the units.
d. Click or tap Insert Expression and select Custom Expression.
e. In the Expression field, enter the expression you found is Step 3a. Be sure to use quotes around the column name(s) in your expression (e.g., "X Stream"). **Note**: Adjust your expression as needed to ensure the calculated exit velocity is positive.
f. Click or tap Apply.

4. Display a graph of exit velocity *vs*. time.
   a. Click or tap the vertical axis label to open the Plot Manager. Turn on Exit Velocity and turn off all other columns.
   b. Describe how the velocity of the water leaving the tank changes as the fluid level in the tank decreases.

5. Use Bernoulli's principle to find the theoretical exit velocity of the water.

$$\mathrm{P}_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = \mathrm{P}_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2$$

   a. Consider the conservation statement for Bernoulli's principle shown above. Comparing a point at the surface of the water column with a point at the exit hole, identify values in the expression that you know and values at the two points that would be the same.
   b. Substitute those values into the equation and simplify the equation so that it shows the relationship between the height of the water column and the exit speed of the water leaving the hole.
   c. Solve your equation from Step 5b for the velocity of the water at the exit hole.
   **Note**: This velocity is the theoretical velocity the water stream would leave the exit hole, according to Torricelli's theorem.

6. Create a calculated column for the theoretical exit velocity of the water:
   a. Click or tap Column Options, ⋯, for the Y Level column and choose Add Calculated Column.
   b. Enter **Exit Vel-Theor** as the name and **m/s** as the units.
   c. Click or tap Insert expression and select Custom Expression.
   d. In the Expression field, enter the expression found in Step 5c. Be sure to use quotes around the column name(s) in your expression (e.g., "Y Level").
   e. Click or tap Apply.

7. Compare the theoretical velocity of the water to the value you calculated using the kinematics equations.
   a. View the data table and compare the exit velocities determined by the trajectory of the water stream with the theoretical exit velocities. How closely do they match?
   b. Click or tap the vertical axis label to open the Plot Manager. Turn on Exit Velocity and Exit Vel-Theor, and turn off all other columns. Do you see any trends in the difference between these two values?

8. Compare the two exit velocities by plotting one against the other.
   a. On the graph, click or tap the horizontal axis label to open the Plot Manager. Turn on Exit Velocity to change the graph to plot Exit Vel-Theor *vs*. Exit Velocity.
   b. Click or tap the vertical axis label to open the Plot Manager. Turn on only Exit Vel-Theor.
   c. Click or tap Graph Tools, , choose Apply Curve Fit, and apply a linear curve fit.
   d. If the Theoretical Exit Velocity and Exit Velocity data were identical, the slope of the linear curve fit would be 1. How close is the slope to 1? How do you account for the discrepancy?

9. If you wanted to place a cup in the correct place to catch water falling in a parabolic trajectory from a filled tank with a small hole in the side, should you use Torricelli's Theorem and kinematics principles to determine where to place the cup? Explain.

# Extension

Does the cross sectional area of the tank enter into this evaluation? What effect, if any, would having a very narrow tank versus a very wide tank have on the exit velocity of the water in the tank?