  Graphical Analysis 4

Determining g on an Incline

(Motion Detector)

During the early part of the seventeenth century, Galileo experimentally examined the concept of acceleration. One of his goals was to learn more about freely falling objects. Unfortunately, his timing devices were not precise enough to allow him to study free fall directly. Therefore, he decided to limit the acceleration by using fluids, inclined planes, and pendulums. In this experiment, you will see how the acceleration of a rolling ball or cart depends on the incline angle. Then, you will use your data to extrapolate to the acceleration on a vertical “incline;” that is, the acceleration of a cart dropped in free fall.

If the angle of an incline with the horizontal is small, a cart rolling down the incline moves slowly and can be easily timed. Using time and position data, it is possible to calculate the acceleration of the cart. When the angle of the incline is increased, the acceleration also increases. The acceleration is directly proportional to the sine of the incline angle, θ. A graph of acceleration versus sin(θ) can be extrapolated to a point where the value of sin(θ) is 1. When sin(θ) is 1, the angle of the incline is 90°. This is equivalent to free fall. The acceleration during free fall can then be determined from the graph.

Galileo was able to measure acceleration only for small angles. You will collect similar data. Can these data be used in extrapolation to determine a useful value of g, the acceleration of free fall? We will see how valid this extrapolation can be. Rather than measuring time, as Galileo did, you will use a Motion Detector to determine the acceleration. You will make quantitative measurements of the motion of a cart rolling down inclines of various small angles. From these measurements, you should be able to decide for yourself whether an extrapolation to large angles is valid.

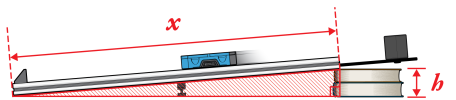


Figure 1

objectives

* Measure the velocity and acceleration of a cart rolling down an incline.
* Determine the mathematical relationship between the angle of an incline and the acceleration of a cart rolling down the incline.
* Determine the value of free fall acceleration, g, by using an extrapolation on the acceleration vs. sine of track angle graph.
* Determine if an extrapolation of the acceleration vs. sine of track angle is valid.

Materials

Chromebook, computer, or mobile device

Graphical Analysis 4 app

Go Direct Motion

Vernier Dynamics Track

Motion Detector Bracket

Adjustable End Stop

Vernier Dynamics Cart

hard ball, approximately 8 cm diameter

rubber ball, similar size

meter stick

books

Preliminary questions

1. One of the timing devices Galileo used was his pulse. Drop a rubber ball from a height of about 2 m and try to determine how many pulse beats elapsed before it hits the ground. What was the timing problem that Galileo encountered?
2. Now measure the time it takes for the rubber ball to fall 2 m, using a watch or clock with a second hand or seconds display. Did the results improve substantially?
3. Roll the hard ball down an incline that makes an angle of about 10° with the horizontal. First use your pulse and then your watch or clock to measure the time of descent.
4. Do you think that during Galileo’s day it was possible to get useful data for any of these experiments? Why?

Procedure

1. Set up the motion detector and Graphical Analysis.
   1. Launch Graphical Analysis.
   2. Connect the motion detector to your Chromebook, computer, or mobile device. Click or tap Sensor Channels.
   3. Select the check box for Motion (cart). Click or tap Done.
   4. To display a graph of velocity vs. time, click or tap the y-axis label and select only Velocity.
2. Set up the equipment and place a single book under one end of the Dynamics Track so that it forms a small angle with the horizontal (see Figure 1). Adjust the points of contact of the two ends of the incline so that the distance, x, in Figure 1, is between 1 and 2 m.
3. Place the motion detector at the top of the incline. Position it so the Dynamics Cart will never be closer than 0.15 m.
4. Hold the cart on the incline about 0.25 m from the motion detector.
5. Click or tap Collect to start data collection; release the cart after the first data point appears on the screen. Quickly move your hand out of the motion detector path. You may have to adjust the position and aim of the motion detector several times before you get it right. Adjust and repeat this step until you get a good run showing an approximately constant slope on the velocity vs. time graph during the rolling of the cart.
6. Fit a straight line to a portion of your data.
   1. Select the data in the linear region of the velocity graph.
   2. Click or tap Graph Tools, , for the velocity vs. time graph and choose Apply Curve Fit.
   3. Select Linear as the curve fit and click or tap Apply.
   4. Record the slope of the fitted line (the acceleration) in the data table.
7. Repeat Steps 4–6 two more times. Note: The previous data set is automatically saved.
8. Measure the length of the incline, x, which is the distance between the two contact points of the incline (see Figure 1). Record the length in your data table.
9. Measure the height, h, of the book(s). Record the height in your data table. These last two measurements will be used to determine the angle of the incline.
10. Raise the incline by placing another book under the end. Adjust the books so that the distance, x, is the same as the previous reading.
11. Repeat Steps 4–9 for the new incline.
12. Repeat Steps 4–10 for 3, 4, and 5 books.

Data Table

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Number of books | Height of books, h    (m) | Length of incline, x  (m) | sin(θ) | Acceleration | | | Average acceleration (m/s2) |
| Trial 1 (m/s2) | Trial 2 (m/s2) | Trial 3 (m/s2) |
| 1 |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |

Analysis

1. Using trigonometry and your values of x and h in the data table, calculate the sine of the incline angle for each height. Note that x is the hypotenuse of a right triangle.
2. Plot a graph of the average acceleration (y-axis) vs. sin(θ). To leave room for extrapolation, carry the horizontal axis out to sin(θ) = 1 (one)
3. Draw a best-fit line by hand or use the curve fit tool and determine the slope. The slope can be used to determine the acceleration of the cart on an incline of any angle.
4. On the graph, carry the fitted line out to sin(90°) = 1 on the horizontal axis and read the value of the acceleration.[[1]](#footnote-1)
5. How well does the extrapolated value agree with the accepted value of free-fall acceleration (g = 9.8 m/s2)?
6. Discuss the validity of extrapolating the acceleration value to an angle of 90°.

Extensions

1. Use the motion detector or video analysis to measure the actual free fall of a ball. Compare the results of your extrapolation with the measurement for free fall.
2. Compare your results in this experiment with other measurements of g. For example, use the experiment, "Picket Fence Free Fall," in this book.
3. Investigate how the value of g varies around the world. For example, how does altitude affect the value of g? What other factors cause this acceleration to vary from place to place? How much can g vary at a school in the mountains compared to a school at sea level?

1. Notice that extrapolating to the y value at the x = 1 point is equivalent to using the slope of the fitted line. [↑](#footnote-ref-1)