  Graphical Analysis 30

Newton’s Law of Cooling

A container of hot water at temperature, T, placed in a room of lower temperature Troom, will result in an exchange of heat from the hot water to the room. The water will eventually cool to the same temperature as the room. You observe this cooling process every time you wait for a hot drink to cool. In this experiment, you will examine the cooling of hot water, with the goal of creating a model that describes the process. You can also predict the time it takes for the hot water to cool to room temperature.

Isaac Newton modeled the cooling process by assuming that the rate at which thermal energy moved from one body to another is proportional (by a constant, k) to the difference in temperature between the two bodies, Tdiff. In the case of a sample of water cooling in room temperature air



From this simple assumption, he showed that the temperature change is exponential in time and can be predicted by



where T0 is the initial temperature difference. Exponential changes are common in science. Systems in which a rate of change is proportional to the changing quantity show exponential behavior.

To complete this experiment in a short time, you will use a small quantity of hot water at a temperature about 30°C above room temperature. A temperature probe will record the water’s temperature as it cools.

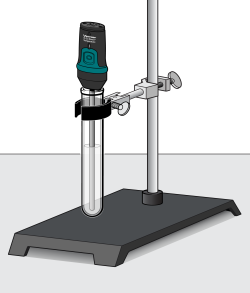


Figure 1

objectives

* Record the cooling process of hot water.
* Test Newton’s law of cooling using your collected water temperature data.
* Use Newton’s law of cooling to predict the temperature of cooling water at any time.

Materials

Chromebook, computer, or mobile device

Graphical Analysis 4 app

Go Direct Temperature

utility clamp

ring stand

20 × 150 mm test tube

20 mL of hot water (50–60°C)

PRELIMINARY QUESTION

A coffee drinker is faced with the following dilemma. She is not going to drink her hot coffee with cream for ten minutes but wants it to still be as hot as possible. Is it better to immediately add the room-temperature cream, stir the coffee, and let it sit for ten minutes, or is it better to let the coffee sit for ten minutes and then add and stir in the cream? Which results in a higher temperature after ten minutes?

Procedure

1. Launch Graphical Analysis. Connect Go Direct Temperature to your Chromebook, computer, or mobile device.
2. Set up the data-collection Mode
   1. Click or tap Mode to open Data Collection Settings.
   2. Change Time Units to min.
   3. Change Rate to 20 samples/min and End Collection to 20 min. Click or tap Done.
3. Determine the room temperature. To do this, hold the temperature probe in the air away from heat sources and sunlight. Monitor the temperature and record the value in your data table.
4. Position the temperature probe at the top of the ring stand using the utility clamp. You will lower it down in a later step.
5. Obtain 20 mL of hot water in your test tube. You may be able to get water this hot from a hot water faucet. If necessary, heat water to this temperature. Secure the test tube to the ring stand, as shown in Figure 1.
6. Lower the temperature probe into the water. Position the tip of the temperature probe in the middle of the water; it should not touch the sides or the bottom of the test tube.
7. Wait about 10 seconds for the temperature probe to reach the temperature of the water. Collect your cooling data.
   1. Click or tap Collect to start data collection.
   2. Collect data for 20 minutes, or until the temperature of the water is within 5° of room temperature. If this occurs, you can click or tap Stop to stop data collection early.
8. When data collection is complete, a graph of temperature vs. time will be displayed.
   1. Click or tap the graph to examine the temperature and time values. Note: You can also adjust the Examine line by dragging the line.
   2. Sketch or print the graph.

Data table

|  |  |
| --- | --- |
| Room temperature (°C) |  |

|  |  |
| --- | --- |
| a |  |
| b |  |
| c |  |

Analysis

1. Fit the exponential function a exp(–cx)+b to your data.
   1. Click or tap Graph Tools, , and choose Apply Curve Fit.
   2. Select Natural Exponent as the Curve Fit. Click or tap Apply.
   3. Record the fit parameters a, b, and c in your data table.
   4. Close the Natural Exponent curve fit box.
2. Newton’s law of cooling was given above as



where Tdiff is the difference between the temperature of the water sample, T, and room temperature, Troom. T0 is the initial temperature difference; in other words, the value of Tdiff at t = 0.



Rearranging these variables, we have



Match the variables x, y, a, b, and c in the fitted equation to the terms T, Troom, k, and t in the expression above. What are the units of a, b, and c? Compare your value for b to the room temperature you recorded earlier. During data collection, was the sensor ever at room temperature?

1. When t = 0, what is the value of e–kt?
2. When t is very large, what is the value of the temperature difference? What is the temperature of the water at this time?
3. What could you do to your experimental apparatus to decrease the value of k in another run? What quantity does k measure?
4. Use your equation to calculate the temperature after 800 seconds. Compare your calculated value with the actual data value.
5. Use your equation to predict the time it takes the water to reach a temperature 1°C above room temperature.
6. If the starting temperature difference is cut in half, does it take half as long to get to 1°C above room temperature?

Extensions

1. Take data for a longer period of time so that the water cools to nearly room temperature. This may take more than 30 minutes. Does the exponential model still fit the data?
2. Use the temperature probe to perform the experiment described in the Preliminary Question. Explain your results in terms of the assumptions Newton made about cooling.
3. Use the temperature probe to experiment with coffee cups made of different materials. Does a drink cool faster in a ceramic cup than in a Styrofoam™ cup? What variables must you hold constant in order to guarantee that the difference in the data is due to the cup? What part of the exponential equation is related to the cup?
4. The mathematical model for the cooling of a liquid can also be used to explain other phenomena in nature. For example, radioactivity and RC circuits behave in a similar fashion. Find other phenomena that are modeled by exponential functions. If possible, make a measurement of the phenomenon in your physics lab.