Terminal Velocity

INTRODUCTION

When evaluating the forces acting on, and motion of, an object moving through the air, we often simplify the situation by excluding air drag. In many cases, it is appropriate to do so—the object may be shaped such that the drag force is negligible. But, when the object in question is large or moving fast, it is harder to justify the exclusion of air drag.

In this experiment, you will examine the motion of a falling object where air drag is significant. Using the Vernier Video Analysis app, you will analyze a video of falling coffee filters. Since these coffee filters have little mass and a relatively large surface area, the effect of air drag is particularly noticeable. You will investigate features of the position *vs*. time and velocity *vs*. time graphs in an effort to model the air drag force.

OBJECTIVES

In this experiment, you will

- Use video analysis techniques to obtain position, velocity, and time data for a falling object that experiences significant air drag.
- Analyze a velocity *vs.* time graph for evidence that the object has reached terminal velocity.
- Evaluate the $\boldsymbol{F}_{drag} = -b\boldsymbol{v}$ model for air drag force.

MATERIALS

Vernier Video Analysis™ app in a web browser on a computer, Chromebook, or mobile device

PRE-LAB INVESTIGATION

Take a flat sheet of paper, hold it about 1.5 m above the floor, and then drop it. As it falls, consider what factors affect its *terminal velocity*, the velocity at which the downward pull of the earth is balanced by upward air drag. Make a list of those factors.

PROCEDURE

1. Launch Vernier Video Analysis. Scroll through the list of sample videos and choose “Coffee Filters.” Play the movie once, so you can be familiar with the motion. You will analyze the video in the following steps.

2. Make the movie window large enough to easily view the falling coffee filters. There are two ways to do this: 1) Click or tap the divider between the video and the other elements on the screen, and drag the divider to the right, or 2) use View, , to remove the graph and data table from view.

3. Set the scale in the video.
   a. Click or tap Scale, . A scale bar and a set of axes will appear on the video. For this experiment, leave the origin at the default location or align it with the 120 cm line on the video.
   b. Drag the ends of the scale bar to match the distance markings in the video. One end should be at 0 cm and the other end should be directly below at the 100 cm line (see Figure 1).
   c. Verify that the length and units are set correctly (should be 1 m).

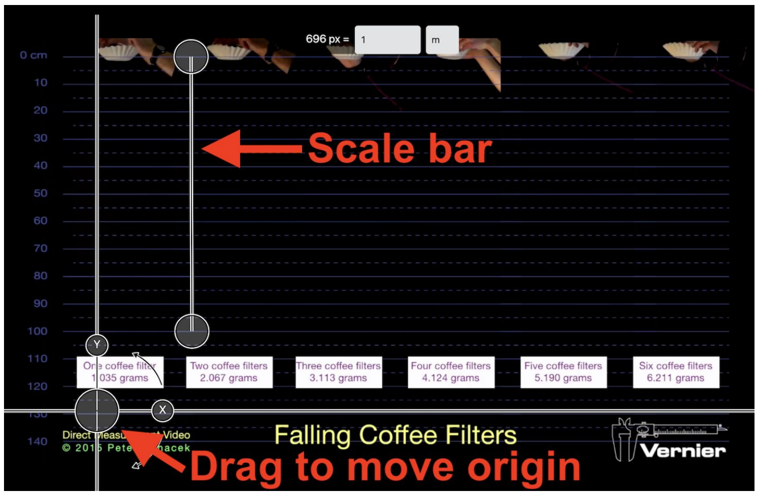

Figure 1

4. Use Step Forward, , and Step Back, , to advance the movie to the frame in which the first coffee filter is clearly released.

5. You will only be marking the position of the first coffee filter for this activity. By default, each time you mark the object's location, the movie advances by just one frame. For this video analysis, it is better if the movie advances more quickly. To change the setting, click or tap Advanced Video Options, . Change the Advance Frame setting to 5 or 10 frames, and then dismiss the window to save your changes.

6. Mark the location of the coffee filter as it moves during the video:
   a. Click or tap Add, .
   b. Decide where on the coffee filter you will mark its location (e.g., the center of the filter, or the bottom-right corner of the filter). **Important**: Be consistent in your marking. Always place the crosshairs in the same place on the coffee filter.
   c. Position the crosshairs at the location on the first coffee filter in the video, and then click or tap to add the first point. **Note**: If you are using a phone or tablet, once you position the crosshairs you can click or tap anywhere in the video frame to mark the point.

d. Continue this process until the coffee filter passes the 120 cm mark. Should you wish to edit a point, click or tap Edit, . This allows you to move or delete a mismarked point by dragging it. **Note**: In order to be sure you are moving the correct dot, you can turn off the trails to hide all the dots except the one in the frame you are viewing (click or tap Trails, , to deactivate the trails).

7. Look at the graph window. If you hid it earlier, use View, , to bring it back. Vernier Video Analysis defaults to display both the *x* and *y* positions of the object as a function of time. For a falling object like the coffee filter, you want to examine the velocity *vs*. time graph of the *y* component, only. To change which data are displayed, click or tap the vertical axis label. Deselect X and Y (the *x* and *y* component of position) and select Y Velocity (the *y* component of velocity).

EVALUATION OF DATA

1. Examine the graph of *y*-velocity *vs.* time. The coffee filter's speed increases in the negative direction until it reaches a somewhat constant velocity. This constant velocity is its terminal velocity. Determine and record the terminal velocity:
   a. Click or tap and drag to select the data points recorded during the terminal velocity motion.
   b. Click or tap Graph Tools, , and select Statistics.
   c. Record the mean value reported in the Statistics window. This average velocity is the coffee filter's terminal velocity.

2. Terminal velocity is reached when the downward pull of the earth is balanced out by the upward air drag. Is the air drag constant over the duration of the coffee filter's fall? What features of the *y*-velocity graph support your conclusion?

3. A simple relationship to describe a changing drag force would be $\boldsymbol{F}_{drag} = -b\boldsymbol{v}$, where b is a drag coefficient and $\boldsymbol{v}$ is the velocity of the object. With this model for the drag force, as the object's velocity increases, so does the drag force (in the opposite direction of the velocity). When $v = v_T$ (terminal velocity), the drag force is great enough that it equals the pull of gravity (mg). Given this relationship, derive an expression for v_T in terms of m, g, and b.

4. Use your expression from the previous step and data from the video to solve for the drag coefficient, b, for the coffee filter. Be sure to include units!

5. Now look at the motion of the coffee filter before it reaches terminal velocity. During the time between its release and reaching terminal velocity, it is speeding up but at a slower and slower rate. Its motion can be described by Newton’s second law:

$$F_{Net} = ma$$

$$bv - mg = ma$$

$$a = \frac{dv}{dt} = \frac{b}{m}v - g$$

   Consider what functions *v(t)* might satisfy the equation above. One possibility is the natural exponential function $v(t) = ae^{ct} + b$ (where *a*, *b*, and *c* are constants), because the derivative of an exponential function is another exponential function. How well does this model fit our *y*-velocity data?

   a. If necessary, close the Statistics window.
   b. Click or tap and drag to select all of the *y*-velocity data points.
   c. Click or tap Graph Tools, , and select Apply Curve Fit.
   d. Choose Natural Exponent from the list of curve fits, and then click or tap Apply.
   e. Take a screenshot of the curve fit and record the fitting parameters a, b, and c.

6. The natural exponential curve fit equation has two terms, constant *b* and ae^{-ct}.
   - How does the equation change overall with time?
   - Which fitting parameter corresponds to the terminal velocity?
   - How does the value of this fitting parameter match the value of the terminal velocity you measured in Step 4?

EXTENSIONS

1. Repeat the data collection and evaluation for another stack of coffee filters in the movie. How well does the model $\boldsymbol{F}_{drag} = -b\boldsymbol{v}$ fit this new data?

2. Select an object that you think will experience a relatively large drag force like the coffee filter does. Use a camera or mobile device to record video of this object falling, and then collect and evaluate the data as you did for the coffee filter. Compare the results of your analyses.