Collision: Physical Pendulum and Cart

INTRODUCTION

You have no doubt explored conservation of momentum during collisions between objects moving in one dimension (linear motion), such as when carts on a track collide. You may have also studied the concept of conservation of angular momentum. Perhaps you conducted a classic experiment involving dropping a disk or ring onto a rotating disk and observing the change in angular velocity. In this activity you are going to explore a collision between a physical pendulum and a cart on a track. The first clearly has angular momentum and the second clearly has linear momentum. But are those momentums equal? In other words, is momentum conserved in a collision involving a rotating rigid object if the object with which it collides moves linearly after the collision?

In the first part of this activity you will investigate the conservation of linear momentum of the physical pendulum and the cart. Then you will explore the conservation of angular momentum of these objects relative to the point of rotation of the pendulum.

OBJECTIVES

In this experiment, you will

- Use video analysis techniques to determine the linear momentum of a cart and a point on a physical pendulum.
- Use video analysis techniques to determine the angular momentum of a physical pendulum.
- Use video analysis tools to determine the angular momentum of a "point object" in 1-dimensional motion.
- Evaluate the conservation of linear and angular momentum as an object in circular motion collides with a stationary object, causing it to move linearly.

MATERIALS

Vernier Video Analysis app in a web browser on a computer, Chromebook, or mobile device
Pendulum-Cart.mp4 video file
Pendulum-Period.mp4 video file

PRE-LAB INVESTIGATION

1. Launch Vernier Video Analysis. Import the "Pendulum-Cart.mp4" movie. Play the movie once or twice to observe the collision between the pendulum and the cart.

2. Based on the video, which of the following things do you think are true? Explain your reasoning.
   - The mass of the physical pendulum must equal the mass of the cart.
   - The horizontal speed of the pendulum just before it collides with the cart is equal to the horizontal speed of the cart after the collision.
   - The product of the mass of the pendulum and its horizontal speed just before it strikes the cart is equal to the product of the mass of the cart and its horizontal speed just after the collision.

3. Identify the information that you need to calculate the angular momentum of the physical pendulum and the linear momentum of the cart. What information can you get from analysis of the video? What information must be provided in order to find these values?

4. Discuss your answers with your classmates.

INVESTIGATION (ABRIDGED INSTRUCTIONS)

1. Add points to the video to show the motion of the lowest mass of the physical pendulum from the moment it is released until the frame in which it collides with the cart. You should mark a point in every frame. If you mark additional points, click or tap Edit, , and delete the points beyond the collision.

2. Reset the video back to several frames before the collision occurs. Click or tap Objects, , and select **+ADD NEW OBJECT**.

3. Add points to the video of the cart starting when the collision occurs. The orange bumper on the cart provides a good target.

4. Click or tap Objects, . Now click or tap Object 1 options, , and change the name to **Pend**. Click or tap < Back and likewise change Object 2 to **Cart**.

5. Set the scale using the meter stick shown in the video. Adjust the origin so the horizontal axis is aligned with (or parallel to) the track and the vertical axis passes through the point of rotation of the pendulum.

6. Adjust the view to show the video and one graph. Set up the graph to show *x* and *y* velocity vs. time of both the pendulum and the cart.

7. Save your video analysis file.

PROCEDURE (SCAFFOLDED INSTRUCTIONS)

1. Make the movie window large enough to easily view the motion of the pendulum and cart. There are two ways to do this: 1) Click or tap the divider between the video and the other elements on the screen, and drag the divider to the right, or 2) use View, , to remove the graph and data table from view.

2. Click or tap Reset Video, , and then use Step Forward, , and Step Back, , to advance the movie to the first frame in which the pendulum is released.

3. Mark the location of the lowest pendulum mass as it moves during the video. **Note**: By default, each time you mark the object's location, the movie advances by just one frame, which is appropriate for this video.
   a. Click or tap Add, .
   b. Click or tap Trails, , to hide all the marked points except the one in the frame you are viewing. This can make it easier to mark and edit points.
   c. Position the crosshairs over the center of the lowest hanging mass, and then click or tap to add the first point. **Note**: If you are using a phone or tablet, once you position the crosshairs you can click or tap anywhere in the video frame to mark the point.
   d. Continue this process until the mass collides with the cart. **Important**: You should mark only one point at the position where the mass is in contact with the cart. If you mark additional points, click or tap Edit, , and delete the points beyond the collision. Turn on trails if necessary..

5. Mark the position of the cart.
   a. Use Step Back , , to move the video back to the frame where the collision occurs. Click or tap Objects, , and select **+ADD NEW OBJECT**.
   b. Click or tap Add, , if it is not selected.
   c. Position the crosshairs on the edge of the orange bumper, and then click or tap to add the first point. **Note**: If you are using a phone or tablet, once you position the crosshairs you can click or tap anywhere in the video frame to mark the point.
   d. Continue marking the bumper of the cart until the end of the video.

6. Change the name of the objects.
   a. Click or tap Objects, , then click or tap Object 1 options, . Change the name to **Pend**. Click or tap **<Back** to save the change.
   b. Click or tap Object 2 options, . Change the name to **Cart**. Click or tap **<Back** to save the change.

7. Set the scale and the origin in the video.
   a. Click or tap System, . You will see new icons appear for Scale and Origin. Scale, , is already selected.
   b. Move one of the scale circles so its center aligns with the left end of the meter stick resting on the track. Move the other scale circle to a convenient mark on the meter stick (70 or 80 cm works well).
   c. Enter the appropriate value for the scale, for example, **0.7** m if you placed the second circle at 70 cm on the meter stick.
   d. Click or tap Origin, .
   e. Adjust the origin so the horizontal axis (*x* axis) is aligned with the track. You may need to rotate the axis to accomplish this.
   f. Place the origin so that the vertical axis passes through the point of rotation of the pendulum.

6. Use View, , to display one graph and the video, and hide the data table.

7. Vernier Video Analysis defaults to display both the *x* and *y* positions of the object as a function of time on the Graph. Click or tap the vertical axis label to open the Plot Manager.

Turn off all the columns. Then, turn on the X Pend Velocity, Y Pend Velocity, X Cart Velocity, and Y Cart Velocity.

8. Save your video analysis file.

ANALYSIS

Part 1 Linear Momentum From Angular Motion?

1. Determine the velocity of the cart after the collision. Given that the mass of the cart is 277 g, calculate the linear momentum of the cart after the collision.

2. Determine the horizontal velocity of the pendulum mass immediately before the collision. The actual mass of the physical pendulum in the video was 281 g.

3. Presume for a moment that momentum is conserved in this collision. Why do you think the calculations show otherwise?

Part 2 Calculating Angular Momentum Of A Physical Pendulum

It should be clear from your work in Part 1 that you cannot simply apply what you know about conservation of linear momentum to systems that also include rotational motion of a physical pendulum. In the next part of this activity, you will use video analysis tools to find the angular momentum of the physical pendulum and again evaluate whether momentum is conserved.

The formula for finding the angular momentum (L) of a physical pendulum is $L = I\omega$, where I is the moment of inertia of the pendulum and ω is the angular velocity. The following steps will describe how to determine these values.

Moment of Inertia

In order to determine the moment of inertia of the physical pendulum, we need more information than just the mass. For example, if you knew the period of the physical pendulum, an equation that relates the period (T) to moment of inertia (I) could be used to determine I. Consider the equation

$$T = 2\pi\sqrt{I/Mgl}$$

where M is the mass of the pendulum (281 g), g is the gravitational constant, and l is the distance from the point of rotation to the center of mass of the pendulum (17.8 cm for the pendulum in the video).

To determine the period of the pendulum, you will use the movie, Pendulum-Period, that shows the pendulum swinging freely.

4. Determine the period of the pendulum.
   a. Open a new tab in your browser and launch Vernier Video Analysis. **Note**: Leave the Pendulum-Cart tab open; you will use that movie again later on.
   b. Import the "Pendulum-Period.mp4" movie. Play the movie once or twice to become familiar with the motion.
   c. Use the video to determine the period of the pendulum and record this value. Describe how you determined the period. If you marked points and used graphs to determine the period, print or sketch your graphs.
   d. Save your video analysis file for the pendulum period.

5. Use your value for the period to find the moment of inertia (I) for the pendulum. Record this value.

Angular Velocity

The angular velocity (ω) for the pendulum can be found from the Pendulum-Cart video. You will need to make a few adjustments to the coordinate system. **Note**: In the Video Analysis app, angular velocity is referred to as θ Velocity.

6. Determine the angular velocity (θ Pend Velocity) for the pendulum.
   a. Return to your original video analysis file from Part 1. Click or tap System, .
   b. Click or tap Origin, , and select Polar.
   c. Confirm that the origin coincides with the point of rotation of the pendulum.
   d. Click or tap View, , to display just one graph and video. If necessary, adjust the divider between the graph and video so you can see both clearly.
   e. Click or tap the vertical axis label to open the Plot Manager. Turn on θ Pend Velocity (the angular velocity of the pendulum) and turn off all other plots.
   f. Identify the angular velocity at the point of impact with the cart.

7. Using the pendulum's moment of inertia (Step 5) and angular velocity (Step 6), calculate the angular momentum of the pendulum just as it strikes the cart.

Part 3 Calculating Angular Momentum Of The Cart

It may seem odd to think of the cart as having *angular* momentum as it travels down the track. Imagine watching from the perspective of the point of rotation of the pendulum. As you watch the pendulum "fall" (rotate), you turn your head to follow the motion. This is as you would expect. Now continue following the cart as it moves down the track. Once again, you have to rotate your head to watch the cart move down the track. Based on this perspective, the cart does have rotational motion, just as the pendulum had. However, unlike the pendulum, the cart's distance to the point of rotation is not constant.

The angular momentum of an object moving with a linear velocity relative to a specific fixed point can be found by determining the cross product of the object's radius vector (from the point of rotation) with the linear momentum vector. The formula for determining the magnitude of the angular momentum of the cart is

$$L = rmv \sin \beta$$

where L is the angular momentum, r is the magnitude of the object's radius vector (the distance from the point of rotation to the cart), mv is the magnitude of the object's momentum vector **p**, and β is the angle between the radius vector and the object's momentum vector.

8. Consider Figure 1. Explain how the quantity $r \sin \beta$ always simplifies to the quantity of $r_\perp$ (the component of the radius vector perpendicular to the momentum vector). This represents the closest point of approach of the object to the point of interest for an object moving in a straight line.

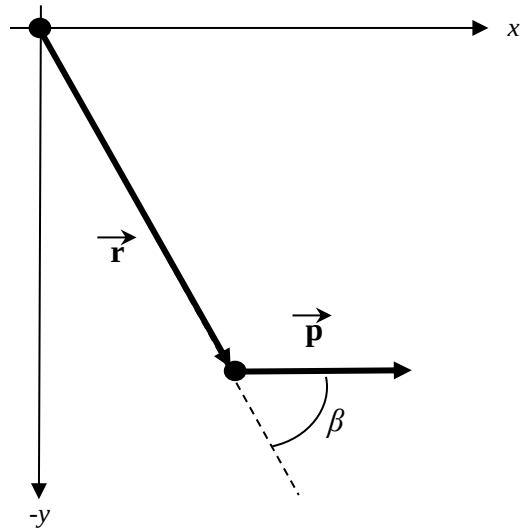

Figure 1 The relationship between the carts radius vector and momentum vectors.

9. Use the Vector tools in Vernier Video Analysis to illustrate the vector relationship shown in Figure 1.
   a. Click or tap Objects, , and select Cart.
   b. Click or tap Add, , if it is not selected, and then click or tap Vectors, .
   c. To display the position vectors, click or tap the Position visibility icon, , until the Resultant, , is displayed. Repeat for Velocity to display the Resultant vector for velocity.
   d. Adjust the Vector Frequency to display the vectors about every 5–10 data points.
   e. Tap Edit, , to dismiss the setup box. Click or tap Reset Video, , to reset the video to the first frame.
   f. Click or tap Trails, , to show the trail of marked points along with the vectors when you play the video. Play the video once or twice with the vectors displayed. This will help you visualize the relationship between the position (**r**) vector and the momentum vector (**p**) since the velocity vector determines the direction of the momentum vector.

10. For simplicity, we will use $r \sin \beta$ (i.e., "$r_\perp$") to calculate the value of the angular momentum of the cart. Use information from the video and your data to find $r_\perp$ for the cart. Explain how you found this value.

11. Determine the angular momentum using $r_\perp$, the peak velocity of the cart after the collision, and the mass of the cart (277 g). **Hint**: $L = rmv \sin \beta = mv\, r\sin \beta = mvr_\perp$

12. Compare the angular momentum of the physical pendulum and the angular momentum of the cart. Can you support an argument for conservation of angular momentum? Explain.

EXTENSIONS

1. Use the Video Analysis tools to determine the magnitude of the angular momentum of the cart at any point as it rolls on the track. Consider these guidelines as you create a calculated column for angular momentum of the cart:

   i. In Video Analysis app, the angle of rotation, θ, is measured from the coordinate system's +X axis and has a value between $0 \leq \theta < 2\pi$ radians.
   ii. When the horizontal axis of the coordinate system is aligned with the track in the video, the cart's momentum vector is parallel to the +X axis of the coordinate system.
   iii. You can use Figure 2 and notes i and ii above to determine the relationship between angles θ and β. **Note**: Angles are measured in radians.
   iv. The mathematical relationship between R Cart and Y Cart could be used in an expression for finding the magnitude of the angular momentum.

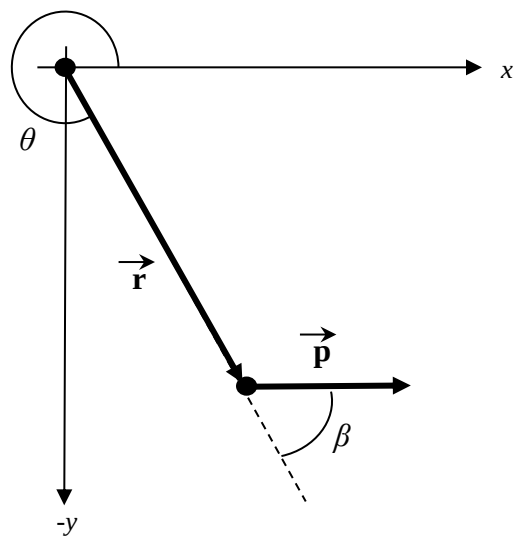

Figure 2

   a. What expression did you use to calculate the magnitude of the cart's angular momentum?
   b. Is the magnitude of the cart's angular momentum constant as it rolls down the track? Would you expect it to be (assuming momentum is conserved)? Explain your reasoning.

2. In this activity, the pendulum collided with the cart approximately at the end of the pendulum arm. If the collision had occurred at the center of mass of the pendulum, answer the following questions:
   a. What effect will this have on the angular momentum of the physical pendulum just as it strikes the cart?
   b. What effect will this have on the angular momentum of the cart after the collision? Explain your reasoning.
   c. The center of mass of the physical pendulum is 17.8 cm from the point of rotation. Is this enough information to be able to calculate the speed of the cart after the collision? If it is, calculate the speed. If it is not, identify any additional information needed to be able to calculate the speed.

3. Create an experiment that allows you to explore the relationship from Extension 2 and report on your results.